The Expansion Option values the flexibility to expand from a current existing state to a larger or expanded state. Therefore, an existing state or condition must first be present in order to use the expansion option. That is, there must be a base case on which to expand. If there is no base case state, then the simple Execution Option (calculated using the simple Call Option) is more appropriate, where the issue at hand is whether to execute a project immediately or to defer execution.

As an example, suppose a growth firm has a static valuation of future profitability using a DCF model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) that is found to be $400 million (PV Asset). Using Monte Carlo simulation, you calculate the implied Volatility of the logarithmic returns on the assets based on the projected future cash flows to be 35 percent. The Risk-Free Rate on a riskless asset (five-year U.S. Treasury note with zero coupons) for the next five years is found to be 7 percent.

Further suppose that the firm has the option to expand and double its operations by acquiring its competitor for a sum of $250 million (Implementation Cost) at any time over the next five years (Maturity). What is the total value of this firm, assuming that you account for this expansion option? The results in Figure 10.11 indicate that the strategic project value is $638.73 (using a 10-step lattice), which means that the expansion option value is $88.73M. This result is obtained because the NPV of executing immediately is $400M · 2 − $250M, or $550M. Thus, $638.73M less $550M is $88.73M, the value of the ability to defer and to wait and see before executing the expansion option. The example file used is Expansion American and European Option.

Increase the dividend rate to, say, 2 percent and notice that both the American and European Expansion Options are now worth less, and that the American Expansion Option is worth more than the European Expansion Option by virtue of the American Option’s ability for early execution (Figure 10.12). The dividend rate implies that the cost of waiting to expand, to defer and not execute, the opportunity cost of waiting on executing the option, and the cost of holding the option, is high, then the ability to defer reduces. In addition, increase the Dividend Rate to 4.9% and see that the binomial lattice’s Custom Option result reverts to $550, (the static, expand-now scenario), indicating that the option is worthless (Figure 10.13). This result means if the cost-of-waiting as a proportion of the asset value (as measured by the dividend rate) is too high, then execute now and stop wasting time deferring the expansion decision! Of course this decision can be reversed if the volatility is significant enough to compensate for the cost of waiting. That is,
it might be worth something to wait and see if the uncertainty is too high even if the cost to wait is high.

Other applications of this option simply abound! To illustrate, following are some additional quick examples of the contraction option and some additional sample exercises.

Suppose a pharmaceutical firm is thinking of developing a new type of insulin that can be inhaled and the drug will be absorbed directly into the bloodstream. A novel and honorable idea. Imagine what this means to diabetics who no longer need painful and frequent injections. The problem is, this new type of insulin requires a brand new development effort but if the uncertainties of the market, competition, drug development, and FDA approval are high, perhaps a base insulin drug that can be ingested is first developed. The ingestible version is a required precursor to the inhaled version. The pharmaceutical firm can decide to either take the risk and fast track development
into the inhaled version or buy an option to defer, to first wait and see if the ingestible version works. If this precursor works, then the firm has the option to expand into the inhaled version. How much should the firm be willing to spend on performing additional tests on the precursor and under what circumstances should the inhaled version be implemented directly? Suppose the intermediate precursor development work yields an NPV of $100M, but at any time within the next two years, an additional $50M can be further invested into the precursor to develop it into the inhaled version, which will triple the NPV. However, after modeling the risk of technical success and uncertainties in the market (competitive threats, sales, and pricing structure), the annualized volatility of the cash flows using the logarithmic present value returns approach comes to 45 percent. Suppose the risk-free rate is 5 percent for the two-year period. Using the SLS, the analysis results yields $254.95M, indicating that the option value to wait and defer is worth more than $4.95M after accounting for the $250M NPV if executing now. In playing with several
scenarios, the breakeven point is found when dividend yield is 1.34 percent. This means that if the cost of waiting (lost net revenues in sales by pursuing the smaller market rather than the larger market, and loss of market share by delaying) exceeds $1.34M per year, then it is not optimal to wait and the pharmaceutical firm should engage in the inhaled version immediately. The loss in returns generated each year does not sufficiently cover the risks incurred.

An oil and gas company is currently deciding on a deep-sea exploration and drilling project. The platform provides an expected NPV of $1,000M. This project is fraught with risks (price of oil and production rate are both uncertain) and the annualized volatility is computed to be 55 percent. The firm is thinking of purchasing an expansion option by spending an additional $10M to build a slightly larger platform that it does not currently need, but if the price of oil is high, or when production rate is low, the firm can execute this expansion option and execute additional drilling to obtain more oil
to sell at the higher price, which will cost another $50M, thereby increasing the NPV by 20 percent. The economic life of this platform is 10 years and the risk-free rate for the corresponding term is 5 percent. Is obtaining this slightly larger platform worth it? Using the SLS, the option value is worth $27.12M when applying a 100-step lattice. Therefore, the option cost of $10M is worth it. However, this expansion option will not be worth it if annual dividends exceed 0.75 percent or $7.5M a year—this is the annual net revenues lost by waiting and not drilling as a percentage of the base case NPV.

Figure 10.14 shows a Bermudan Expansion Option with certain vesting and blackout steps, while Figure 10.15 shows a Customized Expansion Option to account for the expansion factor changing over time. Of course other flavors of customizing the expansion option exist, including changing the implementation cost to expand, and so forth.

**FIGURE 10.14** Bermudan Expansion Option
Exercise: Option to Expand

Suppose a growth firm has a static valuation of future profitability using a DCF model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $400 million. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 35 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 7 percent. Suppose that the firm has the option to expand and double its operations by acquiring its competitor for a sum of $250 million at any time over the next five years. What is the total value of this firm assuming you account for this expansion option?

You decide to use a closed-form approximation of an American call option as a benchmark because the option to expand the firm’s operations can be exercised at any time up to the expiration date. You also decide to confirm
the value of the closed-form analysis with a binomial lattice calculation. Do the following exercises, answering the questions that are posed:

1. Solve the expansion option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the software.
2. Rerun the expansion option problem using the software for 100 steps, 300 steps, and 1,000 steps. What are your observations?
3. Show how you would use the Closed-Form American Approximation Model to estimate and benchmark the results from an expansion option. How comparable are the results?
4. Show the different levels of expansion factors but still yielding the same expanded asset value of $800. Explain your observations of why, when the expansion value changes, the Black-Scholes and Closed-Form American Approximation models are insufficient to capture the fluctuation in value.
   a. Use an expansion factor of 2.00 and an asset value of $400.00 (yielding an expanded asset value of $800).
   b. Use an expansion factor of 1.25 and an asset value of $640.00 (yielding an expanded asset value of $800).
   c. Use an expansion factor of 1.50 and an asset value of $533.34 (yielding an expanded asset value of $800).
   d. Use an expansion factor of 1.75 and an asset value of $457.14 (yielding an expanded asset value of $800).
5. Add a dividend yield and see what happens. Explain your findings.
   a. What happens when the dividend yield equals or exceeds the risk-free rate?
   b. What happens to the accuracy of closed-form solutions like the Black-Scholes and Closed-Form American Approximation Model models for use as benchmarks?
6. What happens to the decision to expand if a dividend yield exists? Now suppose that although the firm has an annualized volatility of 35 percent, the competitor has a volatility of 45 percent. This means that the expansion factor of this option changes over time, comparable to the volatilities. In addition, suppose the implementation cost is a constant 120 percent of the existing firm’s asset value at any point in time. Show how this problem can be solved using the Multiple Asset SLS. Is there option value in such a situation?

**CONTRACTION, EXPANSION, AND ABANDONMENT OPTION**

The Contraction, Expansion, and Abandonment Option applies when a firm has three competing and mutually exclusive options on a single project