

strategy is developed and implemented in the enterprise. The ultimate goal of Phase 3 is to identify risks to the enterprise and develop a protection strategy to mitigate the highest priority risks. The following two processes comprise Phase 3:

- Process 7: Conduct Multidimensional Risk Analysis. This process analyzes the assets, threats, and vulnerability information identified in Phases 1 and 2 using intrusion scenarios to produce a set of risks to the enterprise. The risk attributes of impact and probability are estimated and then used to prioritize the risks.
- Process 8: Develop Protection Strategy. This process develops the protection strategy by identifying candidate mitigation strategies and then selecting the appropriate ones based on factors such as cost and available resources. This process also develops a comprehensive security risk management plan for implementing the protection strategy and managing risks on a continual basis.
- Phase 4: This phase is used to develop customized training programs by analyzing the assets, threats, and vulnerability information in the context of intrusion scenarios, so that an organization can begin to understand what information is at risk. With this understanding, it can create and implement a protection strategy designed to reduce the overall risk exposure of its information assets. The following process comprises a single subprocess:
  - Process 9: Develop Security Knowledge Gap Strategy. This process combines all the above information and industry-specific issues to tailor a training program that “fits” the organization and its current situation to meet current best practices as well as due diligence in its security practices.

## **CASE STUDY: BASEL II CREDIT RISK AND MARKET RISK**

**Analytical Techniques for Modeling Probability of Default, Loss Given Default, Economic Capital, Value at Risk, Portfolio Optimized Value at Risk, Interest Rates and Yield Curve, Delta-Gamma Hedging, Floating and Fixed Rates, Foreign Exchange Risk Hedging, Volatility, Commodity Prices, and Over-the-Counter Exotic Options**

*This case study is written by the author based on consulting projects that he had performed on banks globally, and the case illustrations apply the Risk Simulator, Modeling Toolkit, and Real Options SLS software applications. For more details on some of these applications, please see two of the*

*author's books, Advanced Analytical Models (John Wiley & Sons, 2008) and The Banker's Handbook on Credit Risk: Implementing Basel II (Elsevier Science Academic Press, 2008). This is only meant to be a case study illustrating sample implementations at banks using risk analytic methods and the discussions are not meant to be step-by-step instructions.*

With the new Basel II Accord, internationally active banks are now required to compute their own risk capital requirements using the internal ratings-based (IRB) approach. Not only is adequate risk capital analysis important as a compliance obligation, it provides banks with the ability to optimize their capital through the ability to compute and allocate risks, perform performance measurements, execute strategic decisions, increase competitiveness, and enhance profitability. This case discusses the various *scientific risk management* approaches required to implement an IRB method, as well as the step-by-step models and methodologies in implementing and valuing economic capital, Value at Risk (VaR), probability of default, and loss given default, the key ingredients required in an IRB approach, through the use of advanced analytics such as Monte Carlo and historical risk simulation, portfolio optimization, stochastic forecasting, and options analysis. This case shows the use of Risk Simulator and the Modeling Toolkit (Basel II Toolkit) software in computing and calibrating these critical input parameters. Instead of dwelling on theory or revamping what has already been written many times over, this case focuses solely on the practical modeling applications of the key ingredients to the Basel II Accord.

To follow along with the analyses in this case, we assume that the reader already has Risk Simulator, Real Options SLS, and the Basel II Modeling Toolkit installed, and is somewhat familiar with the basic functions of each software program. If not, please refer to [www.realoptionsvaluation.com](http://www.realoptionsvaluation.com) (click on the *Downloads* link or use the enclosed DVD) and watch the getting started videos, read some of the getting started case studies, or install the latest trial versions of these software programs and their extended licenses. You can download and install the demo version of Modeling Toolkit from the web site and use the password: "heteroskedasticity" when prompted. There is no trial version available; only demo versions or permanent and fully-functional versions are currently available.

### **Probability of Default**

Probability of default measures the degree of likelihood that the borrower of a loan or debt (the obligor) will be unable to make the necessary scheduled repayments on the debt, thereby defaulting on the debt. Should the obligor be unable to pay, the debt is in default, and the lenders of the debt have legal

avenues to attempt a recovery of the debt, or at least partial repayment of the entire debt. The higher the default probability a lender estimates a borrower to have, the higher the interest rate the lender will charge the borrower as compensation for bearing the higher default risk.

Probability of default models are categorized as *structural* or *empirical*. Structural models look at a borrower's ability to pay based on market data such as equity prices, market and book values of asset and liabilities, as well as the volatility of these variables, and hence are used predominantly to estimate the probability of default of *companies* and *countries*, most applicable within the areas of commercial and industrial banking. In contrast, empirical models or credit scoring models are used to quantitatively determine the probability that a loan or loan holder will default, where the loan holder is an individual, by looking at historical portfolios of loans held, where individual characteristics are assessed (e.g., age, educational level, debt to income ratio, and other variables), making this second approach more applicable to the retail banking sector.

**Structural Models of Probability of Default** Probability of default models is a category of models that assesses the likelihood of default by an obligor. They differ from regular credit scoring models in several ways. First of all, credit scoring models are usually applied to smaller credits—individuals or small businesses whereas default models are applied to larger credits—corporation or countries. Credit scoring models are largely statistical, regressing instances of default against various risk indicators, such as an obligor's income, home renter or owner status, years at a job, educational level, and debt to income ratio, something that will be shown later in this case. Structural default models in contrast directly model the default process, and are typically calibrated to market variables, such as the obligor's stock price, asset value, book value of debt, or the credit spread on its bonds. Default models have many applications within financial institutions. They are used to support credit analysis and for finding the probability that a firm will default, to value counterparty credit risk limits, or to apply financial engineering techniques in developing credit derivatives or other credit instruments.

The example illustrated next uses the Merton probability of default model. This model is used to solve the probability of default of a publicly traded company with equity and debt holdings, and accounting for its volatilities in the market (Figure 14.78). This model is currently used by KMV and Moody's to perform credit risk analysis. This approach assumes that the book value of asset and asset volatility are unknown and solved in the model, and that the company is relatively stable and the growth rate of the company's assets are stable over time (e.g., not in start-up mode).

|    | A | B | C | D | E | F | G | H | I | J | K |
|----|---|---|---|---|---|---|---|---|---|---|---|
| 1  |   |   |   |   |   |   |   |   |   |   |   |
| 2  |   |   |   |   |   |   |   |   |   |   |   |
| 3  |   |   |   |   |   |   |   |   |   |   |   |
| 4  |   |   |   |   |   |   |   |   |   |   |   |
| 5  |   |   |   |   |   |   |   |   |   |   |   |
| 6  |   |   |   |   |   |   |   |   |   |   |   |
| 7  |   |   |   |   |   |   |   |   |   |   |   |
| 8  |   |   |   |   |   |   |   |   |   |   |   |
| 9  |   |   |   |   |   |   |   |   |   |   |   |
| 10 |   |   |   |   |   |   |   |   |   |   |   |
| 11 |   |   |   |   |   |   |   |   |   |   |   |
| 12 |   |   |   |   |   |   |   |   |   |   |   |
| 13 |   |   |   |   |   |   |   |   |   |   |   |
| 14 |   |   |   |   |   |   |   |   |   |   |   |
| 15 |   |   |   |   |   |   |   |   |   |   |   |
| 16 |   |   |   |   |   |   |   |   |   |   |   |
| 17 |   |   |   |   |   |   |   |   |   |   |   |
| 18 |   |   |   |   |   |   |   |   |   |   |   |
| 19 |   |   |   |   |   |   |   |   |   |   |   |
| 20 |   |   |   |   |   |   |   |   |   |   |   |
| 21 |   |   |   |   |   |   |   |   |   |   |   |
| 22 |   |   |   |   |   |   |   |   |   |   |   |
| 23 |   |   |   |   |   |   |   |   |   |   |   |
| 24 |   |   |   |   |   |   |   |   |   |   |   |
| 25 |   |   |   |   |   |   |   |   |   |   |   |
| 26 |   |   |   |   |   |   |   |   |   |   |   |
| 27 |   |   |   |   |   |   |   |   |   |   |   |
| 28 |   |   |   |   |   |   |   |   |   |   |   |
| 29 |   |   |   |   |   |   |   |   |   |   |   |
| 30 |   |   |   |   |   |   |   |   |   |   |   |
| 31 |   |   |   |   |   |   |   |   |   |   |   |
| 32 |   |   |   |   |   |   |   |   |   |   |   |
| 33 |   |   |   |   |   |   |   |   |   |   |   |
| 34 |   |   |   |   |   |   |   |   |   |   |   |
| 35 |   |   |   |   |   |   |   |   |   |   |   |
| 36 |   |   |   |   |   |   |   |   |   |   |   |
| 37 |   |   |   |   |   |   |   |   |   |   |   |
| 38 |   |   |   |   |   |   |   |   |   |   |   |
| 39 |   |   |   |   |   |   |   |   |   |   |   |
| 40 |   |   |   |   |   |   |   |   |   |   |   |
| 41 |   |   |   |   |   |   |   |   |   |   |   |
| 42 |   |   |   |   |   |   |   |   |   |   |   |
| 43 |   |   |   |   |   |   |   |   |   |   |   |
| 44 |   |   |   |   |   |   |   |   |   |   |   |
| 45 |   |   |   |   |   |   |   |   |   |   |   |

**DEFAULT PROBABILITY (EXTERNAL MARKET APPROACH)**

This model is used to solve the probability of default of a publicly traded company with equity and debt holdings, and accounting for its volatilities in the market. This model is currently used by KMV and Moody's to perform credit risk analysis. This approach assumes that the book value of asset and asset volatility are unknown and solved.

**STEP ONE:**

Available market and corporate data stating that we have:

|                                 |          |               |
|---------------------------------|----------|---------------|
| Market Value Equity             | \$3,000  | (in millions) |
| Market Equity Volatility        | 46.62%   | (annualized)  |
| Book Value Liabilities and Debt | \$10,000 | (in millions) |
| Risk-free Spot Rate             | 5.00%    |               |
| Maturity of Debt                | 1.00     |               |

Inputs in the real options model:

|                      |          |                                  |           |
|----------------------|----------|----------------------------------|-----------|
|                      | Solved   | Starting                         | Optimized |
| Call Value           | \$2,491  |                                  |           |
| Asset Value*         | \$12,000 | \$12,000                         | \$12,509  |
| Strike Value         | \$10,000 | Using Modeling Toolkit Functions |           |
| Maturity             | 1        |                                  |           |
| Volatility of Asset* | 10.00%   | 10.00%                           | 11.26%    |
| Risk-free Rate       | 5.0%     | Using Modeling Toolkit Functions |           |

Optimization parameters:

|                               |         |
|-------------------------------|---------|
| Call value:                   | \$3,000 |
| Computed value:               | \$2,491 |
| Minimize Absolute Difference: | \$509   |

Optimization Constraints:

|  |        |               |        |
|--|--------|---------------|--------|
| Set value  | 39.65% | to be exactly | 46.62% |
| Set Lower Bounds for Asset Value and Volatility of Asset at \$1 and 1% |        |               |        |

**STEP TWO:**

Default Probability is computed using the Risk Simulator Distribution Analysis tool on:

|                                  |         |
|----------------------------------|---------|
| Default Probability:             | 1.1507% |
| Distance to Default:             | 2.2732  |
| Computed Expected Recovery Rate: | 96.33%  |
| Computed Market Value of Debt:   | 9509.11 |

**FIGURE 14.78** Probability of default model for external public firms.

The model uses several simultaneous equations in options valuation theory coupled with optimization to obtain the implied underlying asset's market value and volatility of the asset in order to compute the probability of default and distance to default for the firm.

**Illustrative Example: Structural Probability of Default Models on Public Firms** It is assumed that at this point, the reader is well versed in running simulations and optimizations in Risk Simulator. The example model used is the *Probability of Default – External Options Model* and can be



accessed through *Modeling Toolkit | Prob of Default | External Options Model (Public Company)*.

To run this model (Figure 14.78), enter in the required inputs such as the market value of equity (obtained from market data on the firm's capitalization, that is, stock price times number of shares outstanding), equity volatility (computed in the Volatility or LPVA worksheets in the model), book value of debt and liabilities (the firm's book value of all debt and liabilities), the risk-free rate (the prevailing country's risk-free interest rate for the same maturity as the debt), and the debt maturity (the debt maturity to be analyzed, or enter 1 for the annual default probability). The comparable option parameters are shown in cells G18 to G23. All these comparable inputs are computed except for Asset Value (the market value of asset) and the Volatility of Asset. You will need to input some rough estimates as a starting point so that the analysis can be run. The rule of thumb is to set the volatility of the asset in G22 to be one-fifth to half of the volatility of equity computed in G10, and the market value of asset (G19) to be approximately the sum of the market value of equity and book value of liabilities and debt (G9 and G11).

Then, an optimization needs to be run in Risk Simulator in order to obtain the desired outputs. To do this, set Asset Value and Volatility of Asset as the decision variables (make them continuous variables with a lower limit of 1% for volatility and \$1 for asset, as both these inputs can only take on positive values). Set cell G29 as the objective to minimize as this is the absolute error value. Finally, the constraint is such that cell H33, the implied volatility in the default model is set to exactly equal the numerical value of the equity volatility in cell G10. Run a static optimization using Risk Simulator.

If the model has a solution, the absolute error value in cell G29 will revert to zero (Figure 14.79). From here, the probability of default (measured in percent) and distance to default (measured in standard deviations) are computed in cells G39 and G41.

Then, using the resulting probability of default, the relevant credit spread required can be determined using the *Credit Analysis – Credit Premium* model or some other credit spread tables (such as using the *Internal Credit Risk Rating* model).

The results indicate that the company has a probability of default at 0.87% with 2.37 standard deviations to default, indicating good creditworthiness (Figure 14.79).

**Illustrative Example: Structural Probability of Default Models on Private Firms** In addition, several other structural models exist for computing the probability of default of a firm. Specific models are used depending on

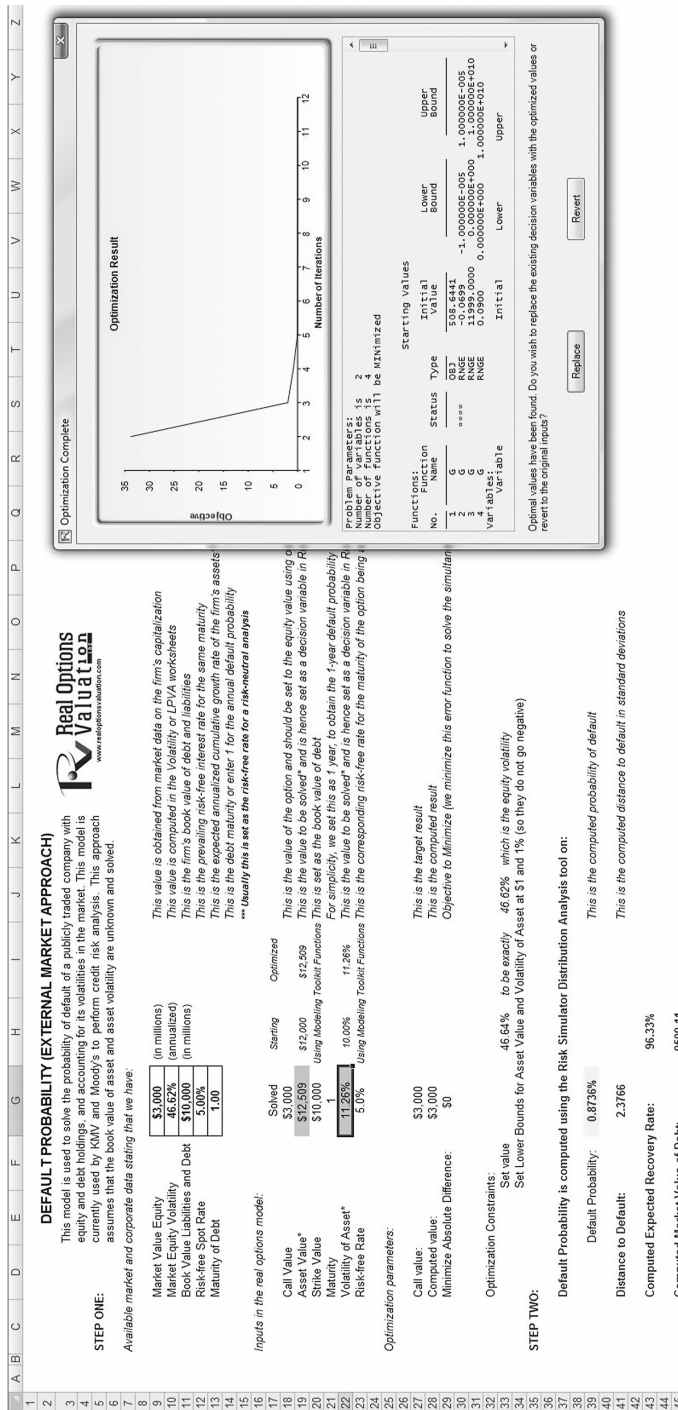


FIGURE 14.79 Optimized model results showing probability of default.

**CREDIT RISK DEFAULT PROBABILITY (OPTIONS APPROACH)**  
VALUING DEFAULT PROBABILITY AND DISTANCE TO DEFAULT BASED ON  
OPTIONS MODELING OF INTERNAL DEBT



**Input Assumptions**

|                     |           |
|---------------------|-----------|
| Asset Book Value    | \$12.0000 |
| Debt Book Value     | \$10.0000 |
| Maturity            | 1.0000    |
| Risk-free Rate      | 7.00%     |
| Volatility of Asset | 10.00%    |

This is the options approach to computing the probability of default and distance to default of a company assuming that the book values of asset and debt are known, as are the asset volatilities and anticipated annual growth rates. If the book value of assets or volatility of assets are not known and the company is publicly traded, use the External Markets model instead. This model assumes these inputs are known or the company is privately held and not traded.

|                        |         |
|------------------------|---------|
| Probability of Default | 0.6695% |
| Distance to Default    | 2.4732  |

Function: B2ProbabilityDefaultMertonI (Asset Value, Strike, Maturity, Riskfree, Asset Volatility)

Function: B2ProbabilityDefaultMertonDefaultDistance(Asset Value, Strike, Maturity, Asset Volatility, Riskfree Rate)

**FIGURE 14.80** Probability of default of a privately held company.

the need and availability of data. In the previous example, the firm is a publicly traded firm, with stock prices and equity volatility that can be readily obtained from the market. In this next example, we assume that the firm is privately held, meaning that there would be no market equity data available. It essentially computes the probability of default or the point of default for the company when its liabilities exceed its assets, given the asset's growth rates and volatility over time (Figure 14.80). It is recommended that before using this model, the previous model on external publicly traded company is first reviewed. Similar methodological parallels exist between these two models, whereby this example builds upon the knowledge and expertise of the previous example.

In Figure 14.80, the example firm with an asset value of \$12M and a book value of debt at \$10M with significant growth rates of its internal assets and low volatility returns a 0.67% probability of default. In addition, instead of relying on the valuation of the firm, external market benchmarks can be used if such data are available. In Figure 14.81, we see that additional input assumptions such as the market fluctuation (market returns and volatility) and relationship (correlation between the market benchmark and the company's assets) are required. The model used is the *Probability of Default – Merton Market Options Model* accessible from *Modeling Toolkit | Prob of Default | Merton Market Options Model (Industry Comparable)*.

**Empirical Models of Probability of Default** As mentioned previously, empirical models of probability of default are used to compute an individual's default probability, applicable within the retail banking arena, where

**MERTON MODEL OF DEBT DEFAULT PROBABILITY**  
VALUING THE PROBABILITY OF DEFAULT BASED ON MARKET RELATIONSHIPS



**Input Assumptions**

|                     |            |
|---------------------|------------|
| Asset Value         | \$100.0000 |
| Debt Value          | \$50.0000  |
| Time to Maturity    | 1.00       |
| Risk-free Rate      | 5.00%      |
| Volatility of Asset | 20.00%     |
| Market Volatility   | 10.00%     |
| Market Return       | 8.00%      |
| Correlation         | 0.00       |

This models the probability of default for both public and private companies using an index or set of comparables (the market), assuming that the company's asset and debt book values are known, as well as the asset's annualized volatility. Based on this volatility and the correlation of the company's assets to the market, we can determine the probability of default.

**Probability of Default** 0.0150%

Function: B2ProbabilityDefaultMertonI (Asset, Debt, Maturity, Riskfree, Asset Volatility, Market Volatility, Market Return, Correlation)

**FIGURE 14.81** Default probability of a privately held entity calibrated to market fluctuations.

empirical or actual historical or comparable data exist on past credit defaults. The dataset in Figure 14.82 represents a sample of several thousand previous loans, credit or debt issues. The data show whether each loan had defaulted or not (0 for no default, and 1 for default), as well as the specifics of each loan applicant's age, education level (1–3 indicating high school, university, or graduate professional education), years with current employer, and so forth. The idea is to model these empirical data to see which variables

**PROBABILITY OF DEFAULT (EMPIRICAL METHOD USING MAXIMUM LIKELIHOOD MODELS ON HISTORICAL DATA)**

| Defaulted | Age | Education Level | Years with Current Employer | Years at Current Address | Household Income (Thousands \$) | Debt to Income Ratio (%) | Credit Card Debt (Thousands \$) | Other Debt (Thousands \$) |
|-----------|-----|-----------------|-----------------------------|--------------------------|---------------------------------|--------------------------|---------------------------------|---------------------------|
| 1         | 41  | 3               | 17                          | 12                       | 176                             | 9.3                      | 11.36                           | 5.01                      |
| 0         | 27  | 1               | 10                          | 6                        | 31                              | 17.3                     | 1.36                            | 4                         |
| 0         | 40  | 1               | 15                          | 14                       | 55                              | 5.5                      | 0.86                            | 2.17                      |
| 0         | 41  | 1               | 15                          | 14                       | 120                             | 2.9                      | 2.66                            | 0.82                      |
| 1         | 24  | 2               | 2                           | 0                        | 28                              | 17.3                     | 1.79                            | 3.06                      |
| 0         | 41  | 2               | 5                           | 5                        | 25                              | 10.2                     | 0.39                            | 2.16                      |
| 0         | 39  | 1               | 20                          | 9                        | 67                              | 30.6                     | 3.83                            | 16.67                     |
| 0         | 43  | 1               | 12                          | 11                       | 38                              | 3.6                      | 0.13                            | 1.24                      |
| 1         | 24  | 1               | 3                           | 4                        | 19                              | 24.4                     | 1.36                            | 3.28                      |
| 0         | 36  | 1               | 0                           | 13                       | 25                              | 19.7                     | 2.78                            | 2.15                      |
| 0         | 27  | 1               | 0                           | 1                        | 16                              | 1.7                      | 0.18                            | 0.09                      |
| 0         | 25  | 1               | 4                           | 0                        | 23                              | 5.2                      | 0.25                            | 0.94                      |
| 0         | 52  | 1               | 24                          | 14                       | 64                              | 10                       | 3.93                            | 2.47                      |
| 0         | 37  | 1               | 6                           | 9                        | 29                              | 16.3                     | 1.72                            | 3.01                      |
| 0         | 48  | 1               | 22                          | 15                       | 100                             | 9.1                      | 3.7                             | 5.4                       |
| 1         | 36  | 2               | 9                           | 6                        | 49                              | 8.6                      | 0.82                            | 3.4                       |
| 1         | 36  | 2               | 13                          | 6                        | 41                              | 16.4                     | 2.92                            | 3.81                      |
| 0         | 43  | 1               | 23                          | 19                       | 72                              | 7.6                      | 1.18                            | 4.29                      |
| 0         | 39  | 1               | 6                           | 9                        | 61                              | 5.7                      | 0.56                            | 2.91                      |
| 0         | 41  | 3               | 0                           | 21                       | 26                              | 1.7                      | 0.1                             | 0.34                      |
| 0         | 39  | 1               | 22                          | 3                        | 52                              | 3.2                      | 1.15                            | 0.51                      |
| 0         | 47  | 1               | 17                          | 21                       | 43                              | 5.6                      | 0.59                            | 1.82                      |
| 0         | 28  | 1               | 3                           | 6                        | 26                              | 10                       | 0.43                            | 2.17                      |
| 0         | 29  | 1               | 8                           | 6                        | 27                              | 9.8                      | 0.4                             | 2.24                      |
| 1         | 21  | 2               | 1                           | 2                        | 16                              | 18                       | 0.24                            | 2.64                      |
| 0         | 25  | 4               | 0                           | 2                        | 32                              | 17.6                     | 2.14                            | 3.49                      |
| 0         | 45  | 2               | 9                           | 26                       | 69                              | 6.7                      | 0.71                            | 3.92                      |
| 0         | 43  | 1               | 25                          | 21                       | 64                              | 16.7                     | 0.95                            | 9.74                      |



The data here represents a sample of several hundred previous loans, credit, or debt issues. The data show whether each loan had defaulted or not, as well as the specifics of each loan applicant's age, education level (1-3 indicating high school, university, or graduate professional education), years with current employer and so forth. The idea is to model these empirical data to see which variables affect the default behavior of individuals, using Risk Simulator's Maximum Likelihood Models. The resulting model will help the bank or credit issuer compute the expected probability of default of an individual credit holder of having specific characteristics.

To run the analysis, select the data on the left or any other data set (include the headers) and make sure that the data have the same length for all variables, without any missing or invalid data. Then, click on Risk Simulator | Forecasting | Maximum Likelihood Models. A sample set of results are provided in the MLE worksheet, complete with detailed instructions on how to compute the expected probability of default of an individual.

**FIGURE 14.82** Empirical analysis of probability of default.

affect the default behavior of individuals, using Risk Simulator's *Maximum Likelihood Estimation* (MLE) tool. The resulting model will help the bank or credit issuer compute the expected probability of default of an individual credit holder having specific characteristics.

#### Illustrative Example on Applying Empirical Models of Probability of Default

The example file is *Probability of Default – Empirical* and can be accessed through *Modeling Toolkit | Prob of Default | Empirical (Individuals)*. To run the analysis, select the data set (include the headers) and make sure that the data have the same length for all variables, without any missing or invalid data points. Then, using Risk Simulator, click on *Risk Simulator | Forecasting | Maximum Likelihood Models*. A sample set of results is provided in the MLE worksheet, complete with detailed instructions on how to compute the expected probability of default of an individual.

The MLE approach applies a modified binary multivariate logistic analysis to model dependent variables to determine the expected probability of success of belonging to a certain group. For instance, given a set of independent variables (e.g., age, income, education level of credit card or mortgage loan holders), we can model the probability of default using MLE. A typical regression model is invalid because the errors are heteroskedastic and non-normal, and the resulting estimated probability forecast will sometimes be above 1 or below 0. MLE analysis handles these problems using an iterative optimization routine. The computed results show the coefficients of the estimated MLE intercept and slopes.<sup>29</sup>

The coefficients estimated are actually the logarithmic odds ratios and cannot be interpreted directly as probabilities. A quick but simple computation is first required. The approach is simple. To estimate the probability of success of belonging to a certain group (e.g., predicting if a debt holder will default given the amount of debt he or she holds), simply compute the estimated Y value using the MLE coefficients. Figure 14.83 illustrates an individual with eight years at a current employer and current address, a low 3% debt to income ratio, and \$2,000 in credit card debt has a log odds ratio of -3.1549. Then, the inverse antilog of the odds ratio is obtained by computing:

$$\frac{\exp(\text{estimated } Y)}{1 + \exp(\text{estimated } Y)} = \frac{\exp(-3.1549)}{1 + \exp(-3.1549)} = 0.0409$$

So, such a person has a 4.09% chance of defaulting on the new debt. Using this probability of default, you can then use the *Credit Analysis – Credit Premium* model to determine the additional credit spread to charge

MLE Results

| Log Likelihood Value            |              | −200.507       |             |         |               |
|---------------------------------|--------------|----------------|-------------|---------|---------------|
| Variable                        | Coefficients | Standard Error | Z-Statistic | p-Value | Sample Inputs |
|                                 | −1.7003      | 0.7512         | −2.2634     | 0.0236  |               |
| Age                             | 0.0279       | 0.0205         | 1.3588      | 0.1742  |               |
| Education Level                 | 0.0728       | 0.1447         | 0.5028      | 0.6151  |               |
| Years with Current Employer     | −0.2528      | 0.0391         | −6.4644     | 0.0000  | 8.000         |
| Years at Current Address        | −0.0952      | 0.0271         | −3.5064     | 0.0005  | 8.000         |
| Household Income (Thousands \$) | 0.0009       | 0.0125         | 0.0754      | 0.9399  |               |
| Debt to Income Ratio (%)        | 0.0750       | 0.0396         | 1.8934      | 0.0583  | 3.000         |
| Credit Card Debt (Thousands \$) | 0.5521       | 0.1324         | 4.1697      | 0.0000  | 2.000         |
| Other Debt (Thousands \$)       | 0.0461       | 0.1005         | 0.4592      | 0.6461  |               |
| Log Odds Ratio                  |              |                |             | −3.1549 |               |
| Default Probability             |              |                |             | 4.09%   |               |

FIGURE 14.83 MLE results.

this person given this default level and the customized cash flows anticipated from this debt holder.

### Loss Given Default and Expected Losses

As shown previously, probability of default is a key parameter for computing credit risk of a portfolio. In fact, the Basel II Accord requires the probability of default, as well as other key parameters such as the loss given default (LGD) and exposure at default (EAD) be reported as well. The reason is that a bank's expected loss is equivalent to:

$$\text{Expected Losses} = (\text{Probability of Default}) \times (\text{Loss Given Default}) \times (\text{Exposure at Default})$$

$$\text{or simply : } EL = PD \times LGD \times EAD$$

PD and LGD are both percentages, whereas EAD is a value. As we have shown how to compute PD in the previous section, we will now revert to some estimations of LGD. There are again several methods used to estimate LGD. The first is through a simple empirical approach where we set  $LGD = 1 - \text{Recovery Rate}$ . That is, whatever is not recovered at default is the

loss at default, computed as the charge off (net of recovery) divided by the outstanding balance:

$$\begin{aligned}\text{LGD} &= 1 - \text{Recovery Rate} \\ \text{or} \\ \text{LGD} &= \frac{\text{Charge Offs (Net of Recovery)}}{\text{Outstanding Balance at Default}}\end{aligned}$$

Therefore, if market data or historical information is available, LGD can be segmented by various market conditions, types of obligor, and other pertinent segmentations (use Risk Simulator's segmentation tool to perform this). LGD can then be readily read off a chart.

A second approach to estimate LGD is more attractive in that if the bank has available information, it can attempt to run some econometric models to create the best fitting model under an ordinary least squares approach. By using this approach, a single model can be determined and calibrated, and this same model can be applied under various conditions, and no data mining is required. However, in most econometric models, a normal-transformation will have to be performed first. Suppose the bank has some historical LGD data (Figure 14.84), the best-fitting distribution can be found using Risk Simulator by first selecting the historical data, and then clicking on *Risk Simulator* | *Tools* | *Distributional Fitting (Single Variable)* to perform the fitting routine. The example's result is a beta distribution for the thousands of LGD values. The p-value can also be evaluated for the goodness of fit of the theoretical distribution (i.e., the higher the p-value, the better the distributional fit, so in this example, the historical LGD fits a beta distribution 81% of the time, indicating a good fit).

Next, using the Distribution Analysis tool in Risk Simulator, obtain the theoretical mean and standard deviation of the fitted distribution (Figure 14.85). Then, transform the LGD variable using the *B2NormalTransform* function in the Modeling Toolkit software. For instance, the value 49.69% will be transformed and normalized to 28.54% (Figure 14.84). Using this newly transformed data set, you can now run some nonlinear econometric models to determine LGD.

The following is a partial list of independent variables that might be significant for a bank, in terms of determining and forecasting the LGD value:

- Debt to capital ratio
- Profit margin
- Revenue

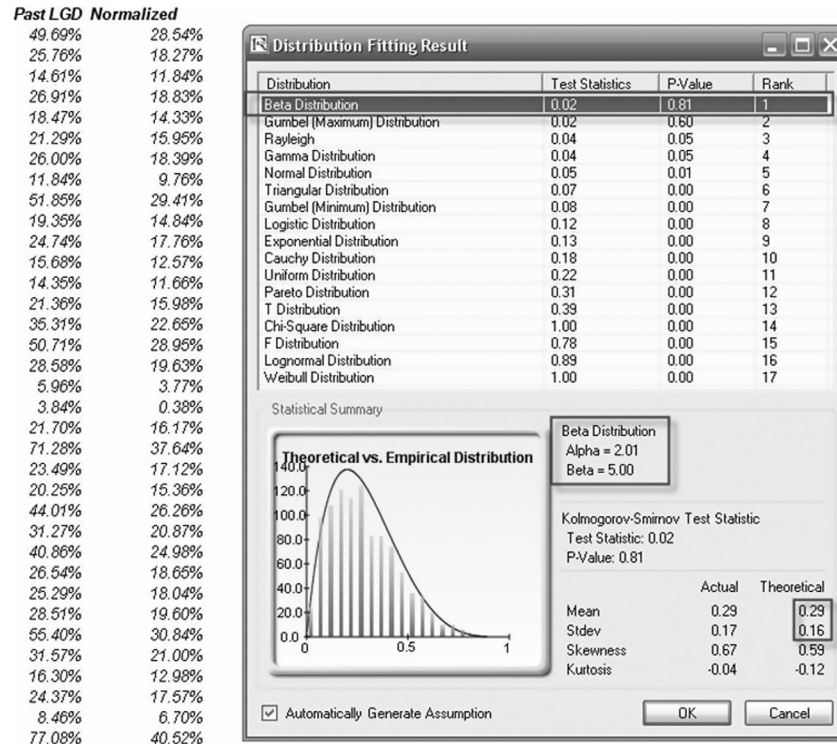


FIGURE 14.84 Fitting historical LGD data.

- Current assets to current liabilities
- Risk rating at default and one year before default
- Industry
- Authorized balance at default
- Collateral value
- Facility type
- Tightness of covenant
- Seniority of debt
- Operating income to sales ratio (and other efficiency ratios)
- Total asset, total net worth, total liabilities

### Economic Capital and Value at Risk

Economic capital is critical to a bank as it links a bank's earnings and returns to risks that are specific to a business line or business opportunity. In addition, these economic capital measurements can be aggregated into a



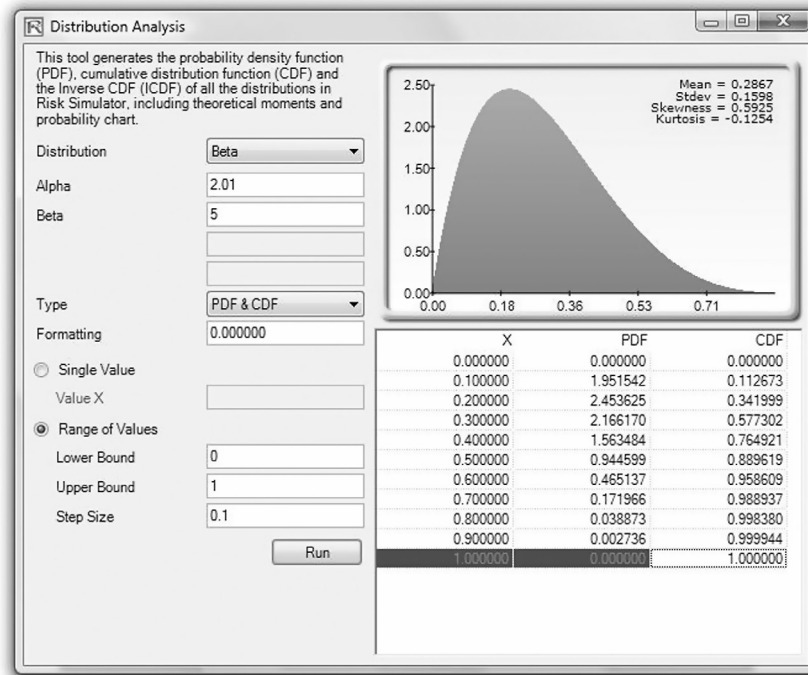


FIGURE 14.85 Distributional analysis tool.

portfolio of holdings. Value at Risk (VaR) is used in trying to understand how the entire organization is affected by the various risks of each holding as aggregated into a portfolio, after accounting for their cross-correlations among various holdings. VaR measures the maximum possible loss given some predefined probability level (e.g., 99.90%) over some holding period or time horizon (e.g., 10 days). The selected probability or confidence interval is typically a decision made by senior management at the bank and reflects the board's risk appetite. Stated another way, we can define the probability level as the bank's desired probability of surviving per year. In addition, the holding period is usually chosen such that it coincides with the time period it takes to liquidate a loss position.

VaR can be computed several ways. Two main families of approaches exist: structural closed-form models and Monte Carlo risk simulation approaches. We will showcase both methods in this case, starting with the structural models.

The second and much more powerful approach is the use of Monte Carlo risk simulation. Instead of simply correlating individual business lines

or assets in the structural models, entire probability distributions can be correlated using more advanced mathematical copulas and simulation algorithms in Monte Carlo simulation methods, by using Risk Simulator. In addition, tens to hundreds of thousands of scenarios can be generated using simulation, providing a very powerful stress-testing mechanism for valuing VaR. In addition, distributional fitting methods are applied to reduce the thousands of data points into their appropriate probability distributions, allowing their modeling to be handled with greater ease.

**Illustrative Example: Structural VaR Models** The first VaR example model shown is the *Value at Risk – Static Covariance Method*, accessible through *Modeling Toolkit | Value at Risk | Static Covariance Method*. This model is used to compute the portfolio's VaR at a given percentile for a specific holding period, after accounting for the cross-correlation effects between the assets (Figure 14.86). The daily volatility is the annualized volatility divided by the square root of trading days per year. Typically, positive correlations tend to carry a higher VaR compared to zero correlation asset mixes, whereas negative correlations reduce the total risk of the portfolio through the diversification effect (Figures 14.86 and 14.87). The approach used is a portfolio VaR with correlated inputs, where the portfolio has multiple asset holdings with different amounts and volatilities. Each asset is also correlated to each other. The covariance or correlation structural model is used to compute the VaR given a holding period or horizon and percentile value (typically 10 days at 99% confidence). Of course, the example only illustrates a few assets or business lines or credit lines for simplicity's sake. Nonetheless, using the functions in the Modeling Toolkit, many more lines, assets, or businesses can be modeled (the function B2VaRCorrelationMethod is used in this example).

**Illustrative Example: VaR Models Using Monte Carlo Risk Simulation** The model used is *Value at Risk – Portfolio Operational and Capital Adequacy* and is accessible through *Modeling Toolkit | Value at Risk | Portfolio Operational and Capital Adequacy*. This model shows how operational risk and credit risk parameters are fitted to statistical distributions and their resulting distributions are modeled in a portfolio of liabilities to determine the Value at Risk (e.g., 99.50th percentile certainty) for the capital requirement under Basel II requirements. It is assumed that the historical data of the operational risk impacts (Historical Data worksheet) are obtained through econometric modeling of the Key Risk Indicators.

The *Distributional Fitting Report* worksheet is a result of running a distributional fitting routine in Risk Simulator to obtain the appropriate

VALUE AT RISK (VARIANCE-COVARIANCE METHOD)

| Asset Allocation | Amount         | Daily Volatility |
|------------------|----------------|------------------|
| Asset A          | \$1,000,000.00 | 1.20%            |
| Asset B          | \$2,000,000.00 | 2.00%            |
| Asset C          | \$3,000,000.00 | 1.89%            |
| Asset D          | \$4,000,000.00 | 3.25%            |
| Asset E          | \$5,000,000.00 | 4.20%            |

| Correlation Matrix | Asset A | Asset B | Asset C | Asset D | Asset E |
|--------------------|---------|---------|---------|---------|---------|
| Asset A            | 1.0000  | 0.1000  | 0.1000  | 0.1000  | 0.1000  |
| Asset B            | 0.1000  | 1.0000  | 0.1000  | 0.1000  | 0.1000  |
| Asset C            | 0.1000  | 0.1000  | 1.0000  | 0.1000  | 0.1000  |
| Asset D            | 0.1000  | 0.1000  | 0.1000  | 1.0000  | 0.1000  |
| Asset E            | 0.1000  | 0.1000  | 0.1000  | 0.1000  | 1.0000  |

|                |        |
|----------------|--------|
| Horizon (Days) | 10     |
| Percentile     | 99.00% |

|                         |                |
|-------------------------|----------------|
| Value at Risk (Daily)   | \$655,915.30   |
| Value at Risk (Horizon) | \$2,074,186.30 |

|   |                |
|---|----------------|
| Daily Value at Risk (Positive Correlations) | \$2,074,186.30 |
| Daily Value at Risk (Zero Correlations)     | \$1,889,345.26 |
| Daily Value at Risk (Negative Correlations) | \$1,684,340.28 |

**FIGURE 14.86** Computing Value at Risk using the structural covariance method.

| Correlation Matrix | Asset A | Asset B | Asset C | Asset D | Asset E |
|--------------------|---------|---------|---------|---------|---------|
| Asset A            | 1.0000  | 0.1000  | 0.1000  | 0.1000  | 0.1000  |
| Asset B            | 0.1000  | 1.0000  | 0.1000  | 0.1000  | 0.1000  |
| Asset C            | 0.1000  | 0.1000  | 1.0000  | 0.1000  | 0.1000  |
| Asset D            | 0.1000  | 0.1000  | 0.1000  | 1.0000  | 0.1000  |
| Asset E            | 0.1000  | 0.1000  | 0.1000  | 0.1000  | 1.0000  |

| Correlation Matrix | Asset A | Asset B | Asset C | Asset D | Asset E |
|--------------------|---------|---------|---------|---------|---------|
| Asset A            | 1.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  |
| Asset B            | 0.0000  | 1.0000  | 0.0000  | 0.0000  | 0.0000  |
| Asset C            | 0.0000  | 0.0000  | 1.0000  | 0.0000  | 0.0000  |
| Asset D            | 0.0000  | 0.0000  | 0.0000  | 1.0000  | 0.0000  |
| Asset E            | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 1.0000  |

| Correlation Matrix | Asset A | Asset B | Asset C | Asset D | Asset E |
|--------------------|---------|---------|---------|---------|---------|
| Asset A            | 1.0000  | -0.1000 | -0.1000 | -0.1000 | -0.1000 |
| Asset B            | -0.1000 | 1.0000  | -0.1000 | -0.1000 | -0.1000 |
| Asset C            | -0.1000 | -0.1000 | 1.0000  | -0.1000 | -0.1000 |
| Asset D            | -0.1000 | -0.1000 | -0.1000 | 1.0000  | -0.1000 |
| Asset E            | -0.1000 | -0.1000 | -0.1000 | -0.1000 | 1.0000  |

**FIGURE 14.87** Different correlation levels.

distribution for the operational risk parameter. Using the resulting distributional parameters, we model each liability's capital requirements within an entire portfolio. Correlations can also be inputted if required between pairs of liabilities or business units. The resulting Monte Carlo simulation results show the Value at Risk or VaR capital requirements.

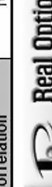
Note that an appropriate empirically based historical VaR cannot be obtained if distributional fitting and risk-based simulations were not first run. Only by running simulations will the VaR be obtained. To perform distributional fitting, follow the steps below:

1. In the *Historical Data* worksheet (Figure 14.88), select the data area (cells C5:L104) and click on *Risk Simulator | Tools | Distributional Fitting (Single Variable)*.
2. Browse through the fitted distributions and select the best-fitting distribution (in this case, the exponential distribution with a particularly high p-value fit, as shown in Figure 14.89) and click OK.
3. You may now set the assumptions on the *Operational Risk Factors* with the exponential distribution (fitted results show  $\text{Lambda} = 1$ ) in the Credit Risk worksheet. Note that the assumptions have already been set for you in advance. You may set it by going to cell F27 and clicking on *Risk Simulator | Set Input Assumption*, selecting *Exponential* distribution and entering 1 for the *Lambda* value and clicking OK. Continue this process for the remaining cells in column F or simply perform a *Risk Simulator Copy* and *Risk Simulator Paste* on the remaining cells:
  - a. Note that since the cells in column F have assumptions set, you will first have to clear them if you wish to reset and copy/paste parameters. You can do so by first selecting cells F28:F126 and clicking on the *Remove Parameter* icon or select *Risk Simulator | Remove Parameter*.
  - b. Then select cell F27, click on the *Risk Simulator Copy* icon or select *Risk Simulator | Copy Parameter*, and then select cells F28:F126 and click on the *Risk Simulator Paste* icon or select *Risk Simulator | Paste Parameter*.
4. Next, additional assumptions can be set such as the probability of default using the Bernoulli distribution (column H) and *Loss Given Default* (column J). Repeat the procedure in Step 3 if you wish to reset the assumptions.
5. Run the simulation by clicking on the *Run* icon or clicking on *Risk Simulator | Run Simulation*.
6. Obtain the Value at Risk by going to the forecast chart once the simulation is done running and selecting *Left-Tail* and typing in 99.50. Hit TAB on the keyboard to enter the confidence value and obtain the VaR of \$25,959 (Figure 14.90).

## Basel II - Credit Risk and Capital Requirement (Portfolio-Based)

This model applies the Basel II requirements on capital adequacy and modeling the operational risk of probability of default on 100 loans as well as the loss given default. These values are fitted based on the bank's historical loss data (Historical Data and Distributional Fitting Report sheets) using Risk Simulator. Then, the relevant historical simulation assumptions are set in this model (Credit Risk sheet) and a Monte Carlo risk-based simulation was run in Risk Simulator to determine the expected capital required and 99.50% Value at Risk (VaR). A simulation has to be run in order to determine the VaR.

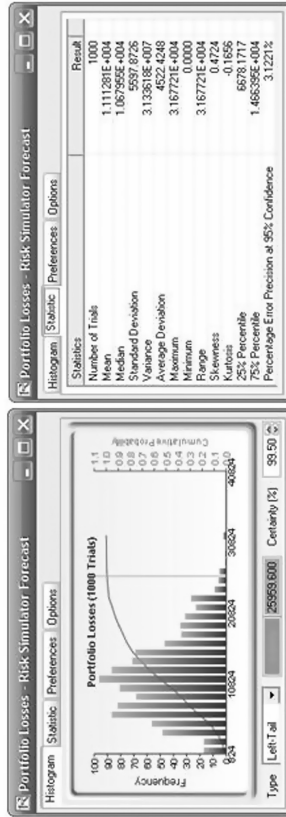
|               |       |              |                         |
|---------------|-------|--------------|-------------------------|
| Market Factor | 2.000 | Rating level | P (Default) - Long term |
| Weighting:    | 50%   | 1            | 0.5%                    |
|               |       | 2            | 1.0%                    |
|               |       | 3            | 1.5%                    |
|               |       | 4            | 2.0%                    |
| Micro         | 50%   | 5            | 2.5%                    |
|               |       | 6            | 3.0%                    |
|               |       | 7            | 5.0%                    |
| Correlation   | 100%  |              |                         |



Real Options  
Valuation

|             |                                 |
|-------------|---------------------------------|
| Static      | Stochastic with Risk-Simulation |
| \$11,734.54 | \$11,112.81                     |
| \$30,888.34 | \$25,959.60                     |

Expected Value of Total Capital  
VaR 99.50% of Total Capital

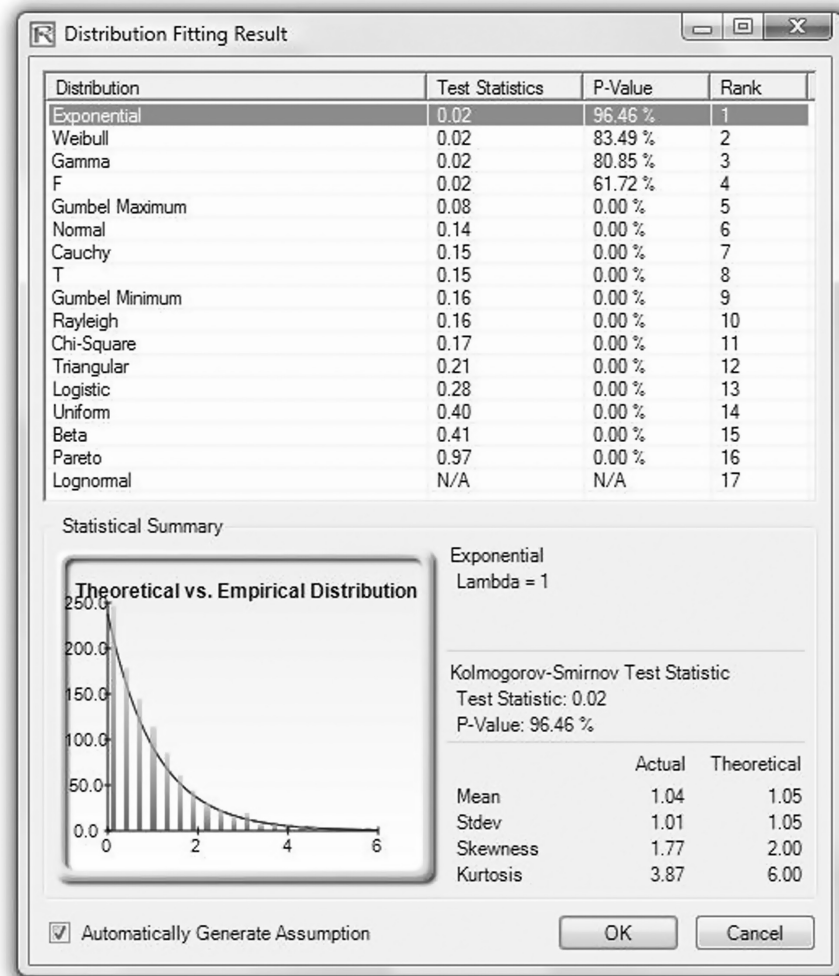


Without running historical simulations, the 99.50% VaR cannot be obtained directly. The only recourse is to apply a theoretical distributional analysis using the fitted distributions' empirical parameters and estimating the theoretical cumulative density function value at 99.50%, and computing the relevant theoretical confidence level. This approach is at best an overestimation of the required capital (thereby requiring too much capital) and at worst, wrong.

|               |             |      |
|---------------|-------------|------|
| Sum           | \$11,734.54 | \$ - |
| Static 99.50% | \$30,888.34 |      |

| Bank loan | Size of loan | Rating grade | P (Default) - Long term |             | Operational Risk Factor | P (Default) - Now |       | Loss Given Default (LGD%) |            | Losses    |            |
|-----------|--------------|--------------|-------------------------|-------------|-------------------------|-------------------|-------|---------------------------|------------|-----------|------------|
|           |              |              | P (Default)             | - Long term |                         | P (Default)       | - Now | Static                    | Stochastic | Static    | Stochastic |
| 1         | \$ 13,274.73 | 5            | 2.5%                    | 2.000       | 2.000                   | 5.00%             | 0     | 30.0%                     | 30.0%      | \$ 199.12 | \$ -       |
| 2         | \$ 14,215.77 | 6            | 3.0%                    | 2.000       | 2.000                   | 6.00%             | 0     | 30.0%                     | 30.0%      | \$ 255.88 | \$ -       |
| 3         | \$ 9,003.59  | 1            | 0.5%                    | 2.000       | 2.000                   | 1.00%             | 0     | 30.0%                     | 30.0%      | \$ 27.01  | \$ -       |
| 4         | \$ 1,324.27  | 3            | 1.5%                    | 2.000       | 2.000                   | 3.00%             | 0     | 30.0%                     | 30.0%      | \$ 11.92  | \$ -       |
| 5         | \$ 11,203.14 | 1            | 0.5%                    | 2.000       | 2.000                   | 1.00%             | 0     | 30.0%                     | 30.0%      | \$ 33.61  | \$ -       |
| 6         | \$ 5,480.61  | 4            | 2.0%                    | 2.000       | 2.000                   | 4.00%             | 0     | 30.0%                     | 30.0%      | \$ 65.77  | \$ -       |
| 7         | \$ 9,853.12  | 5            | 2.5%                    | 2.000       | 2.000                   | 5.00%             | 0     | 30.0%                     | 30.0%      | \$ 147.80 | \$ -       |
| 8         | \$ 12,356.22 | 3            | 1.5%                    | 2.000       | 2.000                   | 3.00%             | 0     | 30.0%                     | 30.0%      | \$ 111.21 | \$ -       |
| 9         | \$ 8,255.80  | 4            | 2.0%                    | 2.000       | 2.000                   | 4.00%             | 0     | 30.0%                     | 30.0%      | \$ 99.07  | \$ -       |
| 10        | \$ 1,662.99  | 2            | 1.0%                    | 2.000       | 2.000                   | 2.00%             | 0     | 30.0%                     | 30.0%      | \$ 9.98   | \$ -       |

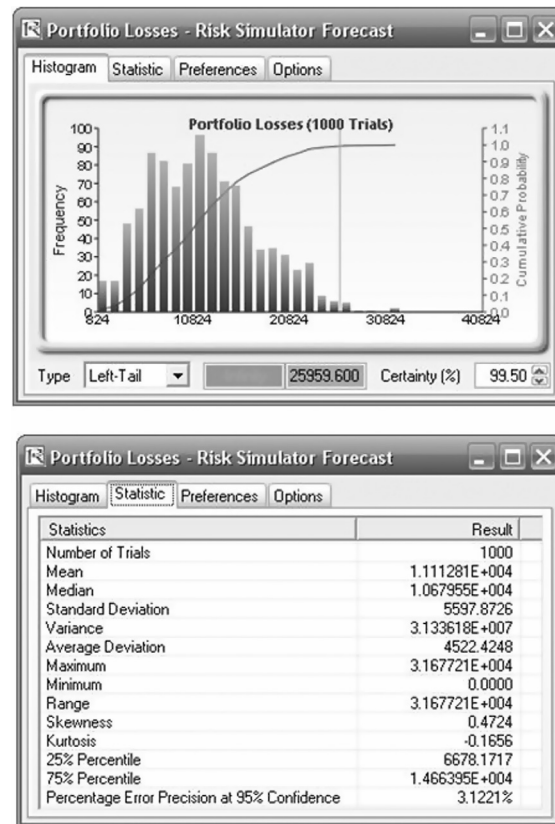
**FIGURE 14.88** Sample historical bank loans.



**FIGURE 14.89** Data fitting results.

Another example on VaR computation is shown next, where the model *Value at Risk – Right Tail Capital Requirements* is used, and available through *Modeling Toolkit | Value at Risk | Right Tail Capital Requirements*.


This model shows the capital requirements per Basel II requirements (99.95th percentile capital adequacy based on a specific holding period's Value at Risk). Without running risk-based historical and Monte Carlo simulation using Risk Simulator, the required capital is \$37.01M



**FIGURE 14.90** Simulated forecast results and the 99.50% Value at Risk value.

(Figure 14.91) as compared to only \$14.00M required using a correlated simulation (Figure 14.92). This is due to the cross-correlations between assets and business lines, and can only be modeled using Risk Simulator. This lower VaR is preferred as banks can now be required to hold less required capital and can reinvest the remaining capital in various profitable ventures, thereby generating higher profits.

1. To run the model, click on *Risk Simulator* | *Run Simulation* (if you had other models open, make sure you first click on *Risk Simulator* | *Change Simulation* | *Profile*, and select the *Tail VaR* profile before starting).
2. When simulation is complete, select *Left-Tail* in the forecast chart and enter in 99.95 in the *Certainty* box and hit *TAB* on the keyboard to obtain the value of \$14.00M Value at Risk for this correlated simulation.



Real Options

Valuation

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TOTAL VALUE AT RISK MODEL (BASEL II REQUIREMENT)

Line of Business

Mean Required Capital

99.95th Percentile

Capital Required

Allocation Weights

Minimum Allowed

Maximum Allowed

Business 1

Business 2

Business 3

Business 4

Business 5

Business 6

Business 7

Business 8

Business 9

Business 10

\$10.50

\$11.12

\$11.77

\$10.77

\$13.49

\$14.24

\$15.60

\$14.95

\$14.15

\$10.08

\$36.52

\$47.52

\$48.99

\$37.34

\$49.52

\$55.59

\$60.24

\$64.69

\$61.02

\$35.37

\$26.01

\$36.39

\$37.22

\$26.56

\$36.03

\$41.35

\$44.64

\$49.74

\$46.87

\$25.29

10.00%

10.00%

10.00%

10.00%

10.00%

10.00%

10.00%

10.00%

10.00%

10.00%

5.00%

5.00%

5.00%

5.00%

5.00%

5.00%

5.00%

5.00%

5.00%

5.00%

15.00%

15.00%

15.00%

15.00%

15.00%

15.00%

15.00%

15.00%

15.00%

15.00%

Portfolio Total

\$12.67

\$49.68

\$37.01

100.00%

Total Capital Required

\$14.00

Correlation Matrix

1

2

3

4

5

6

7

8

9

10

1

2

3

4

5

6

7

8

9

10

-0.20

-0.13

-0.05

0.23

0.00

0.02

0.25

0.36

-0.01

0.35

0.01

0.15

-0.15

0.03

-0.26

-0.60

0.16

0.00

0.00

0.10

0.00

-0.30

0.04

-0.10

0.00

0.00

0.01

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

0.00

FIGURE 14.91 Right-tail VaR model.





**FIGURE 14.92** Simulated results of the portfolio VaR.

3. Note that the assumptions have already been set for you in advance in the model in cells C6:C15. However, you may set it again by going to cell C6 and clicking on *Risk Simulator* | *Set Input Assumption*, selecting your distribution of choice or use the default *Normal Distribution* or perform a distributional fitting on historical data and click OK. Continue this process for the remaining cells in column C. You may also decide to first *Remove Parameters* of these cells in column C and set your own distributions. Further, correlations can be set manually when assumptions are set (Figure 14.93) or by going to *Simulation* | *Edit Correlations* (Figure 14.94) after all the assumptions are set.

If risk simulation was not run, the VaR or economic capital required would have been \$37M, as opposed to only \$14M. All cross-correlations between business lines have been modeled, as are stress and scenario tests, and thousands and thousands of possible iterations are run. Individual risks are now aggregated into a cumulative portfolio level VaR.

**Efficient Portfolio Allocation and Economic Capital VaR** As a side note, by performing portfolio optimization, a portfolio's VaR can actually be reduced. We start by first introducing the concept of stochastic portfolio optimization through an illustrative hands-on example. Then, using this portfolio optimization technique, we apply it to four business lines or assets to compute the VaR or an un-optimized versus an optimized portfolio of assets, and see the difference in computed VaR. You will note that at the end, the optimized portfolio bears less risk and has a lower required economic capital.

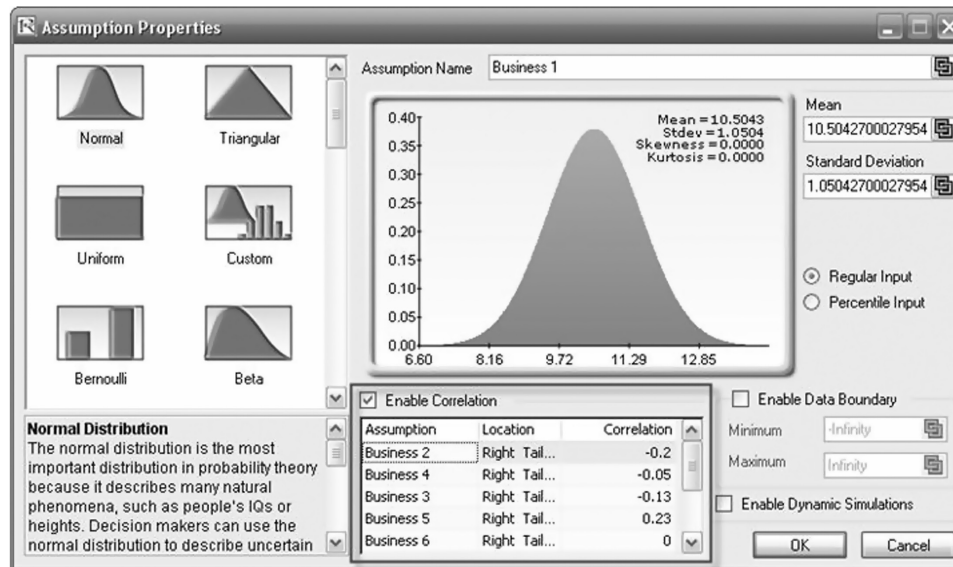


FIGURE 14.93 Setting correlations one at a time.

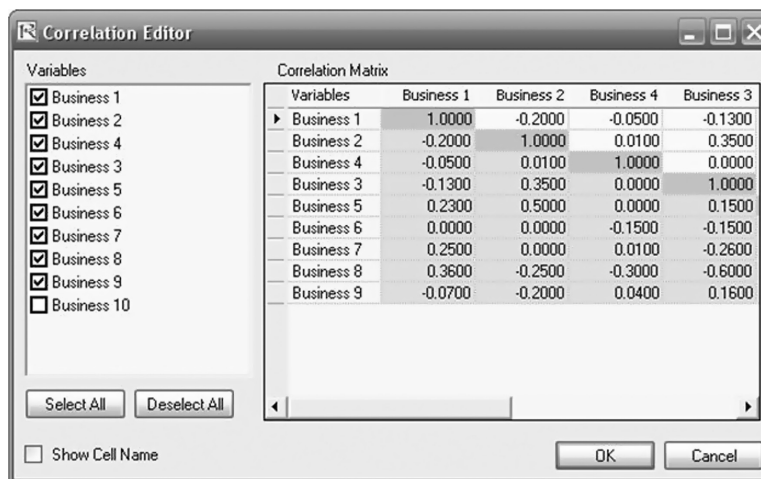


FIGURE 14.94 Setting correlations using the correlation matrix routine.

|    | A | B | C | D | E | F | G |
|----|---|---|---|---|---|---|---|
| 1  |   |   |   |   |   |   |   |
| 2  |   |   |   |   |   |   |   |
| 3  |   |   |   |   |   |   |   |
| 4  |   |   |   |   |   |   |   |
| 5  |   |   |   |   |   |   |   |
| 6  |   |   |   |   |   |   |   |
| 7  |   |   |   |   |   |   |   |
| 8  |   |   |   |   |   |   |   |
| 9  |   |   |   |   |   |   |   |
| 10 |   |   |   |   |   |   |   |
| 11 |   |   |   |   |   |   |   |
| 12 |   |   |   |   |   |   |   |
| 13 |   |   |   |   |   |   |   |
| 14 |   |   |   |   |   |   |   |
| 15 |   |   |   |   |   |   |   |
| 16 |   |   |   |   |   |   |   |
| 17 |   |   |   |   |   |   |   |
| 18 |   |   |   |   |   |   |   |
| 19 |   |   |   |   |   |   |   |
| 20 |   |   |   |   |   |   |   |
| 21 |   |   |   |   |   |   |   |
| 22 |   |   |   |   |   |   |   |
| 23 |   |   |   |   |   |   |   |
| 24 |   |   |   |   |   |   |   |
| 25 |   |   |   |   |   |   |   |
| 26 |   |   |   |   |   |   |   |
| 27 |   |   |   |   |   |   |   |
| 28 |   |   |   |   |   |   |   |
| 29 |   |   |   |   |   |   |   |
| 30 |   |   |   |   |   |   |   |
| 31 |   |   |   |   |   |   |   |
| 32 |   |   |   |   |   |   |   |
| 33 |   |   |   |   |   |   |   |
| 34 |   |   |   |   |   |   |   |
| 35 |   |   |   |   |   |   |   |
| 36 |   |   |   |   |   |   |   |
| 37 |   |   |   |   |   |   |   |
| 38 |   |   |   |   |   |   |   |
| 39 |   |   |   |   |   |   |   |
| 40 |   |   |   |   |   |   |   |
| 41 |   |   |   |   |   |   |   |

**VALUE AT RISK WITH ASSET ALLOCATION OPTIMIZATION MODEL**

**Asset Class Description**

| Annualized Returns   | Volatility Risk | Allocation Weights | Required Minimum Allocation | Required Maximum Allocation |
|----------------------|-----------------|--------------------|-----------------------------|-----------------------------|
| S&P 500              | 7.10%           | 9.80%              | 10.00%                      | 40.00%                      |
| Small Cap            | 9.51%           | 14.35%             | 27.30%                      | 40.00%                      |
| High Yield           | 15.90%          | 22.50%             | 22.70%                      | 40.00%                      |
| Govt Bonds           | 4.50%           | 7.25%              | 40.00%                      | 10.00%                      |
| <b>Total Weight:</b> |                 | <b>100.00%</b>     |                             |                             |

**Correlation Matrix**

| S&P 500    | Small Cap | High Yield | Govt Bonds |
|------------|-----------|------------|------------|
| S&P 500    | 1.0000    | 0.7400     | 0.6500     |
| Small Cap  | 0.7400    | 1.0000     | 0.4200     |
| High Yield | 0.6500    | 0.4200     | 1.0000     |
| Govt Bonds | 0.5500    | 0.3100     | 0.2300     |

**Covariance Matrix**

| S&P 500    | Small Cap | High Yield | Govt Bonds |
|------------|-----------|------------|------------|
| S&P 500    | 0.0096    | 0.0104     | 0.0143     |
| Small Cap  | 0.0104    | 0.0206     | 0.0136     |
| High Yield | 0.0143    | 0.0136     | 0.0506     |
| Govt Bonds | 0.0039    | 0.0032     | 0.0038     |

**Starting Value** \$1,000,000.00  
**Term (Years)** 5.00

**Annualized Return** 8.72%  
**Portfolio Risk** 9.84%  
**Ending Value** \$1,087,151.94

**Profit/Loss** \$87,151.94  
**Return to Risk Ratio** 88.59%

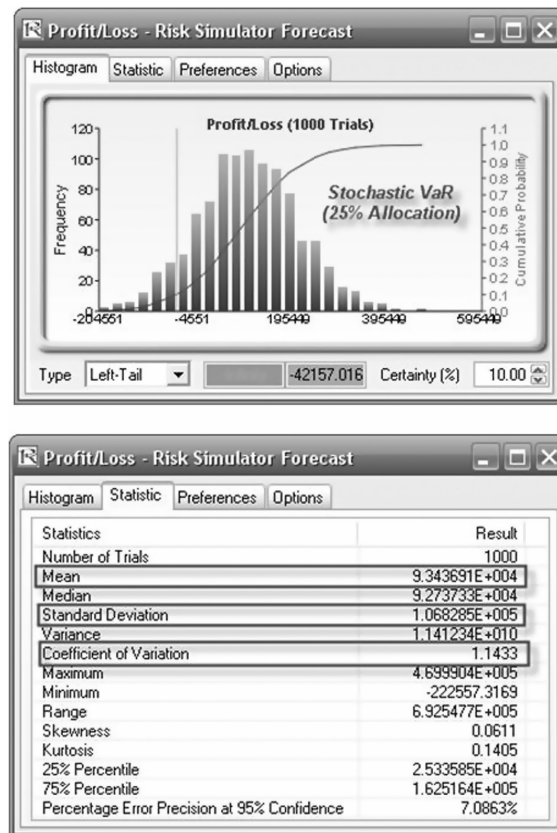
**Specifications of the optimization model:**

**Objective:** Maximize Return to Risk Ratio (E28)  
**Decision Variables:** Allocation Weights (E6:E9)  
**Restrictions on Decision Variables:** Minimum and Maximum Required (F6:G9)  
**Constraints:** Portfolio Total Allocation Weights 100% (E10 is set to 100%)

Notice that the 10% VaR of profit/loss for a simple 25% allocation yields \$42,157 as compared to a VaR of \$35,815 for an optimized portfolio allocation of assets. The VaR is reduced by having a better portfolio allocation. Further, the portfolio risk is also reduced as well the fluctuations of the profit and loss results (coefficient of variability) but at the expense of slightly lower profits (low risk, low returns). This model is already set up and ready to run an optimization analysis.

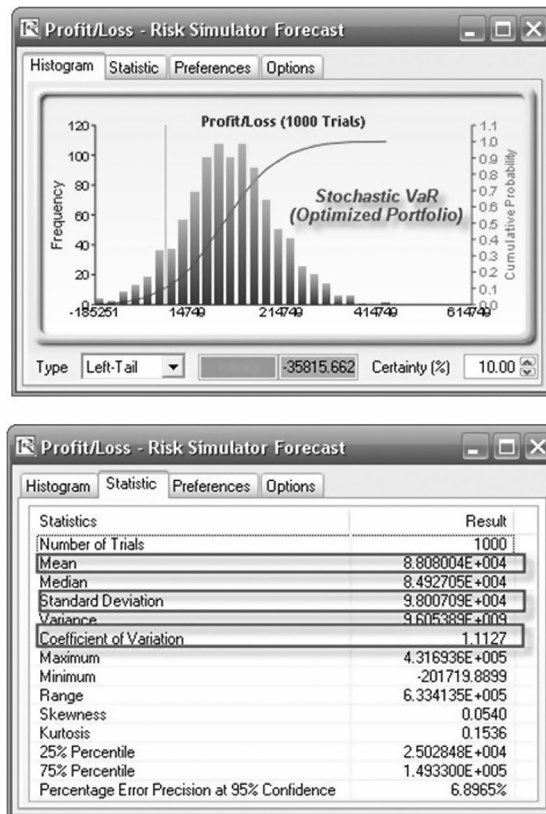
**FIGURE 14.95** Computing Value at Risk (VaR) with simulation.

**Illustrative Example: Portfolio Optimization and the Effects on Portfolio VaR** Now that we understand the concepts of optimized portfolios, let us now see what the effects are on computed economic capital through the use of a correlated portfolio VaR. This model uses Monte Carlo simulation and optimization routines in Risk Simulator to minimize the VaR of a portfolio of assets (Figure 14.95). The file used is *Value at Risk – Optimized and Simulated Portfolio VaR*, which is accessible via *Modeling Toolkit | Value at Risk | Optimized and Simulated Portfolio VaR*. In this example model, we intentionally used only 4 asset classes to illustrate the effects of an optimized portfolio, whereas in real life, we can extend this to cover a multitude of asset classes and business lines. In addition, we now illustrate the use of a left-tail VaR, as opposed to a right-tail VaR, but the concepts are similar.



**FIGURE 14.96** Nonoptimized Value at Risk.

First, simulation is used to determine the 90% left-tail VaR (this means that there is a 10% chance that losses will exceed this VaR for a specified holding period). With an equal allocation of 25% across the 4 asset classes, the VaR is determined using simulation (Figure 14.96). The annualized returns are uncertain and hence simulated. The VaR is then read off the forecast chart. Then, optimization is run to find the best portfolio subject to the 100% allocation across the 4 projects that will maximize the portfolio's bang-for-the-buck (returns to risk ratio). The resulting optimized portfolio is then simulated once again and the new VaR is obtained (Figure 14.97). The VaR of this optimized portfolio is a lot less than the not optimized portfolio. That is, the expected loss is \$35.8M instead of \$42.2M, which means that the bank will have a lower required economic capital if the portfolio of holdings is first optimized.



**FIGURE 14.97** Optimal portfolio's Value at Risk through optimization and simulation.

## Market Risk Analysis

### Illustrative Example: Risk Analysis—Interest Rate Risk

**File Name:** Risk Analysis – Interest Rate Risk

**Location:** Modeling Toolkit | Risk Analysis | Interest Rate Risk

**Brief Description:** Applies duration and convexity measures to account for a bond's sensitivity and how interest rate shifts can affect the new bond price, and how this new bond price can be approximated using these sensitivity measures

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Functions Used:** B2BondPriceDiscrete, B2ModifiedDuration, B2ConvexityDiscrete

### INTEREST RATE RISK

|                                 |                               |                        |
|---------------------------------|-------------------------------|------------------------|
| Face Value                      | \$100.00                      |                        |
| Coupon Rate                     | 5.50%                         |                        |
| Maturity                        | 30.00                         |                        |
| Current Interest Rate           | 5.50%                         |                        |
| Interest Rate Shift             | 0.25%                         |                        |
| <b>Original Bond Price</b>      | \$100.00                      |                        |
| <b>Modified Duration</b>        | 14.5337                       |                        |
| <b>Convexity</b>                | 321.0265                      |                        |
|                                 | <b>Duration and Convexity</b> | <b>Using New Rates</b> |
| <b>New Price After Shift</b>    | \$96.47                       | \$96.46                |
| <b>Price Change After Shift</b> | -3.53%                        | -3.54%                 |

| Cash Flow | Interest Rates | Year | Shifted Interest Rates |
|-----------|----------------|------|------------------------|
| \$5.50    | 5.50%          | 1    | 5.75%                  |
| \$5.50    | 5.50%          | 2    | 5.75%                  |
| \$5.50    | 5.50%          | 3    | 5.75%                  |
| \$5.50    | 5.50%          | 4    | 5.75%                  |
| \$5.50    | 5.50%          | 5    | 5.75%                  |
| \$5.50    | 5.50%          | 6    | 5.75%                  |
| \$5.50    | 5.50%          | 7    | 5.75%                  |
| \$5.50    | 5.50%          | 8    | 5.75%                  |
| \$5.50    | 5.50%          | 9    | 5.75%                  |
| \$5.50    | 5.50%          | 10   | 5.75%                  |

**FIGURE 14.98** Interest rate risk.

Banks selling fixed income products and vehicles need to understand interest rate risks. This model uses duration and convexity to show how fixed income products react under various market conditions. To compare the effects of interest rate and credit risks on fixed income investments, this model uses modified duration and convexity (discrete discounting) to analyze the effects of a change in interest rates on the value of a bond or debt (Figure 14.98).

Duration and convexity are sensitivity measures that describe exposure to parallel shifts in the spot interest rate yield curve, applicable to individual fixed income instruments or entire fixed income portfolios. These sensitivities cannot warn of exposure to more complex movements in the spot curve, including tilts and bends, only parallel shifts. The idea behind duration is simple. Suppose a portfolio has a duration measure of 2.5 years. This means

that the portfolio's value will decline about 2.5% for each 1% increase in interest rates—or rise about 2.5% for each 1% decrease in interest rates. Typically, a bond's duration will be positive but exotic instruments such as mortgage-backed securities may have negative durations, or portfolios that short fixed income instruments or pay fixed for floating on an interest rate swap. Inverse floaters tend to have large positive durations. Their values change significantly for small changes in rates. Highly leveraged fixed-income portfolios tend to have large (positive or negative) durations.

In contrast, convexity summarizes the second-most significant piece of information, or the nonlinear curvature of the yield curve, whereas duration measures the linear or first-approximation sensitivity. Duration and convexity have traditionally been used as tools for immunization or asset-liability management. To avoid exposure to parallel spot curve shifts, an organization (such as an insurance company or defined benefit pension plan) with significant fixed income exposures might perform duration matching by structuring its assets so that their duration matches the duration of its liabilities so the two offset each other. Even more effective (but less frequently practical) is duration-convexity matching, in which assets are structured so that durations and convexities match.

### **Illustrative Example: Risk Hedging—Delta-Gamma Hedging**

**File Name:** Risk Analysis – Delta Gamma Hedge

**Location:** *Modeling Toolkit | Risk Analysis | Delta Gamma Hedge*

**Brief Description:** Sets up a delta-gamma riskless and costless hedge in determining the number of call options to sell and buy, number of common stocks to buy, and the borrowing amount required, to set up a perfect arbitrage-free hedge

**Requirements:** Modeling Toolkit

**Modeling Toolkit Functions Used:** B2DeltaGammaHedgeCallSold,  
B2DeltaGammaHedgeSharesBought,  
B2DeltaGammaHedgeMoneyBorrowed

The Delta-Gamma hedge provides a hedge against larger changes in the asset value (Figure 14.99). This is done by buying some equity shares and a call option, which are funded by borrowing some amount of money and selling a call option at a different strike price. The net amount is a zero sum game, making this hedge completely effective in generating a zero delta and zero gamma for the portfolio. Just like in a delta hedge, where the total portfolio's delta is zero (e.g., to offset a positive delta of some underlying assets, call options are sold to generate sufficient negative delta

### DELTA-GAMMA HEDGE

|   |                   |
|---|-------------------|
| Asset                                   | \$100.00          |
| Strike for Call Sold                    | \$95.00           |
| Strike for Call Bought                  | \$100.00          |
| Maturity for Call Sold                  | 0.50              |
| Maturity for Call Bought                | 0.75              |
| Riskfree                                | 8.00%             |
| Volatility                              | 20.00%            |
| DividendRate                            | 3.00%             |
| <b>Sell Calls</b>                       | <b>\$9.7258</b>   |
| <b>Shares to Buy</b>                    | <b>(\$6.9058)</b> |
| <b>Buy Calls</b>                        | <b>(\$9.1991)</b> |
| <b>Borrow This Amount</b>               | <b>\$6.3791</b>   |
| <b>Delta-Gamma-Neutral Position Sum</b> | <b>\$0.0000</b>   |

**FIGURE 14.99** Delta-gamma hedging.

to completely offset the existing deltas to generate a zero delta portfolio). The problem of delta neutral portfolios is that secondary changes, that is, larger shocks are not hedged. Delta-gamma hedged portfolios, on the contrary, hedge both delta and gamma risk, making it much more expensive to generate. The typical problem with such a hedging vehicle is that in larger quantities, buying and selling additional options or underlying assets may change the market value and prices of the same instruments used to perform the hedge. Therefore, typically, a dynamic hedge or continuously changing hedge portfolios might be required.

### Illustrative Example: Risk Hedging—Delta Hedging

**File Name:** Risk Analysis – Delta Hedge

**Location:** *Modeling Toolkit | Risk Analysis | Delta Hedge*

**Brief Description:** Sets up a delta riskless and costless hedge in determining the number of call options to sell, number of common stocks to buy, and the borrowing amount required to set up a costless hedge

**Requirements:** Modeling Toolkit

**Modeling Toolkit Functions Used:** B2DeltaHedgeCallSold,  
B2DeltaHedgeSharesBought, B2DeltaHedgeMoneyBorrowed

The Delta hedge provides a hedge against small changes in the asset value by buying some equity shares of the asset and financing it through selling a call option and borrowing some money (Figure 14.100). The net should be



### DELTA HEDGE

|                                   |                    |
|-----------------------------------|--------------------|
| Asset                             | \$100.00           |
| Strike                            | \$95.00            |
| Maturity                          | 0.50               |
| Riskfree                          | 8.00%              |
| Volatility                        | 20.00%             |
| DividendRate                      | 3.00%              |
| <b>Sell 1 Call</b>                | <b>\$9.7258</b>    |
| <b>Shares to Buy</b>              | <b>(\$71.8275)</b> |
| <b>Borrow This Amount</b>         | <b>\$62.1018</b>   |
| <b>Delta-Neutral Position Sum</b> | <b>\$0.0000</b>    |

**FIGURE 14.100** Delta hedging.

a zero sum game to provide a hedge where the portfolio's delta is zero. For instance, an investor computes the portfolio delta of some underlying asset, and offsets this delta through buying or selling some additional instruments such that the new instruments will offset the delta of the existing underlying assets. Typically, say an investor holds some stocks or commodity like gold in the long position, creating a positive delta for the asset. To offset this, he or she sells some calls to generate negative delta, such that the amount of the call options sold on the gold is sufficient to offset the delta in the portfolio.

### Illustrative Example: Risk Hedging—Effects of Fixed versus Floating Rates (Swaps)

**File Name:** Risk Hedging – Effects of Fixed versus Floating Rates

**Location:** *Modeling Toolkit | Risk Hedging | Effects of Fixed versus Floating Rates*

**Brief Description:** Sets up various levels of hedging to determine the impact on earnings per share

**Requirements:** Modeling Toolkit

This model illustrates the impact to financial earnings and earnings before interest and taxes (EBIT) on a hedged versus unhedged position (Figure 14.101). The hedge is done through interest rate swap payments. Various scenarios of swaps (different combinations of fixed rate versus floating rate debt are tested and modeled) can be generated in this model to determine the impact to earnings per share (EPS) and other financial metrics. The foreign

### IMPACTS OF FIXED VERSUS FLOATING RATE INTEREST PAYMENTS

#### Assumptions

|                     |             |
|---------------------|-------------|
| EBIT                | \$3,000,000 |
| Shares Outstanding  | \$500,000   |
| Tax Rate            | 40.00%      |
| Total Debt          | \$8,000,000 |
| Fixed Interest Rate | 7.00%       |
| LIBOR               | 6.00%       |
| 10-Year Swap Rate   | 5.00%       |

#### Initial Debt Structure (before swap)

|                                       | Current | 1      | 2      | 3      |
|---------------------------------------|---------|--------|--------|--------|
| % of Total Debt in Fixed-rate Debt    | 50.00%  | 50.00% | 50.00% | 50.00% |
| % of Total Debt in Floating-rate Debt | 50.00%  | 50.00% | 50.00% | 50.00% |

#### Desired Debt Structure (after swap)

|                                       |        |        |         |         |
|---------------------------------------|--------|--------|---------|---------|
| % of Total Debt in Fixed-rate Debt    | 50.00% | 30.00% | 100.00% | 0.00%   |
| % of Total Debt in Floating-rate Debt | 50.00% | 70.00% | 0.00%   | 100.00% |

#### Change in Interest Rates

|       |       |       |       |
|-------|-------|-------|-------|
| 0.00% | 1.00% | 0.50% | 0.10% |
|-------|-------|-------|-------|

#### Financials

|                            |           |           |           |           |
|----------------------------|-----------|-----------|-----------|-----------|
| Fixed-rate Debt            | 7.00%     | 7.00%     | 7.00%     | 7.00%     |
| Floating-rate Debt         | 8.00%     | 9.00%     | 8.50%     | 8.10%     |
| EBIT                       | 3,000,000 | 3,000,000 | 3,000,000 | 3,000,000 |
| Interest Expense           | (600,000) | (672,000) | (560,000) | (648,000) |
| Net Income before Taxes    | 2,400,000 | 2,328,000 | 2,440,000 | 2,352,000 |
| Earnings                   | 1,440,000 | 1,396,800 | 1,464,000 | 1,411,200 |
| EPS                        | 2.8800    | 2.7936    | 2.9280    | 2.8224    |
| Change in Interest Expense |           | 72,000    | (40,000)  | 48,000    |
| Change in Earnings         |           | (43,200)  | 24,000    | (28,800)  |

**FIGURE 14.101** Impacts of an unhedged versus hedged position.

exchange cash flow hedge model (shown next) goes into more detail on the hedging aspects of foreign exchange through the use of risk simulation.

#### Illustrative Example: Risk Hedging—Foreign Exchange Cash Flow Model

**File Name:** Risk Hedging – Foreign Exchange Cash Flow Model

**Location:** *Modeling Toolkit | Risk Hedging | Foreign Exchange Cash Flow Model*

**Brief Description:** This model illustrates how to use Risk Simulator for simulating foreign exchange rates to determine if the value of a hedged fixed exchange rate or floating unhedged rate is worth more

**Requirements:** Modeling Toolkit, Risk Simulator

This is a cash flow model used to illustrate the effects of hedging foreign exchange rates (Figure 14.102). The tornado sensitivity analysis illustrates that foreign exchange rate, or forex, has the highest effects on the profitability of the project (shown in the Excel model). Suppose for the moment that the project undertaken is in a foreign country (FC) and the values obtained are denominated in FC currency, and the parent company is in the United States (U.S.) and requires that the net revenues be repatriated back to the U.S. The question we try to ask here is what is the appropriate forex rate to hedge at and the appropriate costs for that particular rate? Banks will be able to provide your firm with the appropriate pricing structure for various exchange forward rates but by using the model here, we can determine the added value of the hedge, and hence can decide if the value added exceeds the cost to obtain the hedge. This model is already preset for you to run a simulation on.

The *Forex Data* worksheet shows historical exchange rates between the FC and U.S. Dollar. Using these values, we can create a *custom* distribution (we simply used the rounded values in our illustration), which is already preset in this example model.

However, should you wish to replicate creating the simulation model, you can follow the steps below:

1. Start a new profile (*Risk Simulator | New Profile*) and give it an appropriate name.
2. Go to the *Forex Data* worksheet and select the data in cells K6:K490 and click on *Edit | Copy* or *Ctrl + C*.
3. Select an empty cell (e.g., cell K4) and click on *Risk Simulator | Set Input Assumption* and select *Custom Distribution*.
4. Click on *Paste* to paste the data into the custom distribution, then *Update Chart* to view the results on the chart. Then, *File | Save* and save the newly created distribution to your hard drive. Close the set assumption dialog.
5. Go to the *Model* worksheet and select the *Forex* cell (J9) and click on *Risk Simulator | Set Input Assumption*, and choose *Custom*, then, click on *Open* a distribution and select the previously saved custom distribution.
6. You may continue to set assumptions across the entire model, and set the NPV cell (G6) as a forecast (*Risk Simulator | Set Output Forecast*).
7. RUN the simulation with the custom distribution to denote an unhedged position. You can then rerun the simulation but this time, delete the custom distribution (use the *Delete Simulation Parameter* icon and not Excel's delete function or the keyboard's delete key) and enter in the

### Cash Flow Model

|                                 |        |              |             |             |             |             |             |             |             |             |
|---------------------------------|--------|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Base Year                       | 2006   | 2007         | 2008        | 2009        | 2010        | 2011        | 2012        | 2013        | 2014        | 2015        |
| Start Year                      | 2006   | FC 10.00     | FC 10.50    | FC 11.00    | FC 11.50    | FC 12.00    | FC 12.00    | FC 12.00    | FC 12.00    | FC 12.00    |
| Discount Rate                   | 15.00% | FC 12.25     | FC 12.50    | FC 12.75    | FC 13.00    | FC 13.25    | FC 13.25    | FC 13.25    | FC 13.25    | FC 13.25    |
| Private-Risk Discount Rate      | 5.00%  | FC 15.15     | FC 15.30    | FC 15.45    | FC 15.60    | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    |
| Terminal Period Growth Rate     | 2.00%  | 50           | 50          | 50          | 50          | 50          | 50          | 50          | 50          | 50          |
| Tax Rate                        | 40.00% | 35           | 35          | 35          | 35          | 35          | 35          | 35          | 35          | 35          |
|                                 |        | 20           | 20          | 20          | 20          | 20          | 20          | 20          | 20          | 20          |
| Prod A Price                    |        | FC 1,231.75  | FC 1,268.50 | FC 1,305.25 | FC 1,342.00 | FC 1,378.75 | FC 1,378.75 | FC 1,378.75 | FC 1,378.75 | FC 1,378.75 |
| Prod B Price                    |        | FC 184.76    | FC 190.28   | FC 195.79   | FC 201.30   | FC 206.81   | FC 206.81   | FC 206.81   | FC 206.81   | FC 206.81   |
| Prod C Price                    |        | FC 1,046.99  | FC 1,078.23 | FC 1,109.46 | FC 1,140.70 | FC 1,171.94 | FC 1,171.94 | FC 1,171.94 | FC 1,171.94 | FC 1,171.94 |
| Prod A Quantity                 |        | FC 157.50    | FC 157.50   | FC 157.50   | FC 157.50   | FC 157.50   | FC 157.50   | FC 157.50   | FC 157.50   | FC 157.50   |
| Prod B Quantity                 |        | FC 15.75     | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    | FC 15.75    |
| Prod C Quantity                 |        | FC 873.74    | FC 904.98   | FC 936.21   | FC 967.45   | FC 998.69   | FC 998.69   | FC 998.69   | FC 998.69   | FC 998.69   |
| Total Revenues (Local Currency) |        | FC 10.00     | FC 10.00    | FC 10.00    | FC 10.00    | FC 10.00    | FC 10.00    | FC 10.00    | FC 10.00    | FC 10.00    |
| Direct Cost of Goods Sold       |        | FC 3.00      | FC 3.00     | FC 3.00     | FC 3.00     | FC 3.00     | FC 3.00     | FC 3.00     | FC 3.00     | FC 3.00     |
| Gross Profit                    |        | FC 860.74    | FC 891.98   | FC 923.21   | FC 954.45   | FC 985.69   | FC 985.69   | FC 985.69   | FC 985.69   | FC 985.69   |
| Operating Expenses              |        | FC 2.00      | FC 2.00     | FC 2.00     | FC 2.00     | FC 2.00     | FC 2.00     | FC 2.00     | FC 2.00     | FC 2.00     |
| Sales, General and Admin. Costs |        | FC 858.74    | FC 889.98   | FC 921.21   | FC 952.45   | FC 983.69   | FC 983.69   | FC 983.69   | FC 979.69   | FC 978.69   |
| Operating Income (EBITDA)       |        | FC 343.50    | FC 355.99   | FC 388.49   | FC 380.98   | FC 393.08   | FC 392.88   | FC 392.88   | FC 391.88   | FC 391.48   |
| Depreciation                    |        | FC 515.24    | FC 533.99   | FC 552.73   | FC 571.47   | FC 589.21   | FC 589.01   | FC 588.41   | FC 587.21   | FC 587.21   |
| Amortization                    |        | FC 13.00     | FC 13.00    | FC 13.00    | FC 13.00    | FC 13.00    | FC 13.00    | FC 13.00    | FC 13.00    | FC 13.00    |
| EBIT                            |        | FC 0.00      | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     |
| Interest                        |        | FC 0.00      | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     |
| Taxes                           |        | FC 0.00      | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     | FC 0.00     |
| Net Income                      |        | FC 528.24    | FC 546.99   | FC 565.73   | FC 584.47   | FC 603.21   | FC 602.61   | FC 601.41   | FC 600.81   | FC 600.36   |
| Depreciation/Amort              |        |              |             |             |             |             |             |             |             |             |
| Net Working Capital             |        |              |             |             |             |             |             |             |             |             |
| Capital Expenditures            |        |              |             |             |             |             |             |             |             |             |
| Free Cash Flow                  |        |              |             |             |             |             |             |             |             |             |
| Investments                     |        | FC 500.00    |             |             |             |             |             |             |             |             |
| Net Free Cash Flow              |        | -FC 1,105.97 | FC 546.99   | FC 565.73   | FC 584.47   | FC 603.21   | FC 602.61   | FC 601.41   | FC 600.81   | FC 600.36   |

**FIGURE 14.102** Hedging foreign exchange risk cash flow model.

relevant hedged exchange rate, indicating a fixed rate. You may create a report after each simulation to compare the results.

From the sample analysis, we see the following:

|                | Mean<br>(\$000) | Stdev<br>(\$000) | % Confidence (\$000) | CV (%) |
|----------------|-----------------|------------------|----------------------|--------|
| Unhedged       | 2292.82         | 157.94           | 2021 to 2550         | 6.89   |
| Hedged at 0.85 | 2408.81         | 132.63           | 2199 to 2618         | 5.51   |
| Hedged at 0.83 | 2352.13         | 129.51           | 2147 to 2556         | 5.51   |
| Hedged at 0.80 | 2267.12         | 124.83           | 2069 to 2463         | 5.51   |

From this table, several things are evident:

- The higher the hedged exchange rate is, the more profitable the project (e.g., 0.85 USD/FC is worth more than 0.80 USD/FC).
- The relative risk ratio, computed as the coefficient of variation (CV, or the standard deviation divided by the mean), is the same regardless of the exchange rate, as long as it is hedged.
- The CV is lower for hedged positions than unhedged positions, indicating that the relative risk is reduced by hedging.
- It seems that the exchange rate hedge should be above 0.80, such that the hedged position is more profitable than the unhedged.
- In comparing a hedged versus unhedged position, we can determine the amount of money the hedging is worth, for instance, going with a 0.85 USD/FC means that on average the hedge is worth \$115,990 (computed as \$2,408.81 – \$2,292.82 denominated in thousands). This means that as long as the cost of the hedge is less than this amount, it is a good idea to pursue the hedge.

### **Illustrative Example: Risk Hedging—Hedging Foreign Exchange Exposure**

**File Name:** Risk Hedging – Hedging Foreign Exchange Exposure

**Location:** *Modeling Toolkit | Risk Hedging | Hedging Foreign Exchange Exposure*

**Brief Description:** This model illustrates how to use Risk Simulator for simulating foreign exchange rates to determine the value of a hedged currency option position

**Requirements:** Modeling Toolkit, Risk Simulator

**Hedging Foreign Exchange Exposure with Currency Options**

| Months   | Jan        | Feb         | Mar        | April       | May        | June       | July        |
|--|------------|-------------|------------|-------------|------------|------------|-------------|
| FX Spot Rate (HKD/USD)                               | 7.80       | 7.40        | 7.60       | 7.30        | 7.10       | 7.20       | 7.40        |
| FX Strike Rate (HKD/USD)                             | 7.80       | 7.80        | 7.80       | 7.80        | 7.80       | 7.80       | 7.80        |
| Maturity (Years)                                     | 0.5833     | 0.5000      | 0.4167     | 0.3333      | 0.2500     | 0.1667     | 0.0833      |
| Risk Free Rate US                                    | 6.08%      | 6.08%       | 6.08%      | 6.08%       | 6.08%      | 6.08%      | 6.08%       |
| Risk Free Rate HK                                    | 5.06%      | 5.06%       | 5.06%      | 5.06%       | 5.06%      | 5.06%      | 5.06%       |
| Volatility   | 15.00%     | 15.00%      | 15.00%     | 15.00%      | 15.00%     | 15.00%     | 15.00%      |
| Quantity of Options Hedge Position                   | 10,000,000 | 10,000,000  | 10,000,000 | 10,000,000  | 10,000,000 | 10,000,000 | 10,000,000  |
| Currency Put Option Value (HKD/USD)                  | 0.3229     | 0.5191      | 0.3795     | 0.5533      | 0.7012     | 0.6034     | 0.4102      |
| Market Value of Hedge                                | 3,229,135  | 5,191,009   | 3,794,813  | 5,532,845   | 7,012,229  | 6,034,435  | 4,102,320   |
| Intrinsic Value                                      | 0          | 4,000,000   | 2,000,000  | 5,000,000   | 7,000,000  | 6,000,000  | 4,000,000   |
| Time Value   | 3,229,135  | 1,191,009   | 1,794,813  | 532,845     | 12,229     | 34,435     | 102,320     |
| <b>FINANCIAL STATEMENTS IMPACTS - MARK TO MARKET</b> |            |             |            |             |            |            |             |
| Balance Sheet (in 000's)                             | Jan        | Feb         | Mar        | April       | May        | June       | July        |
| Option Contract                                      | 3,229,135  | 5,191,009   | 3,794,813  | 5,532,845   | 7,012,229  | 6,034,435  | 4,102,320   |
| Other Comp Income (SE)                               |            | 4,000,000   | 2,000,000  | 5,000,000   | 7,000,000  | 6,000,000  | 4,000,000   |
| <b>Income Statement (in 000's)</b>                   |            |             |            |             |            |            |             |
| Hedge Effectiveness gain or loss per period          |            | (2,038,126) | 603,805    | (1,261,969) | (520,615)  | 22,206     | 67,884      |
| Hedge Effectiveness sum of all periods               |            |             |            |             |            |            | (3,126,616) |
| Market Cost of Hedge (Current Period)                |            |             |            |             |            |            | 3,229,135   |
| Income from Option Exercise                          |            |             |            |             |            |            | 4,000,000   |
| Net Valuation of Hedging                             |            |             |            |             |            |            | 770,865     |
| Income from Hedging                                  |            |             |            |             |            |            | 74,770,865  |
| Income from No Hedge                                 |            |             |            |             |            |            | 74,000,000  |
| Loss Distribution from Hedging                       |            |             |            |             |            |            | 3,229,135   |
| Loss Distribution from No Hedge                      |            |             |            |             |            |            | 4,000,000   |

**FIGURE 14.103** Hedging currency exposures with currency options.

This model is used to simulate possible foreign exchange spot and future prices and the effects on the cash flow statement of a company under a freely floating exchange rate versus using currency options to hedge the foreign exchange exposure (Figure 14.103).

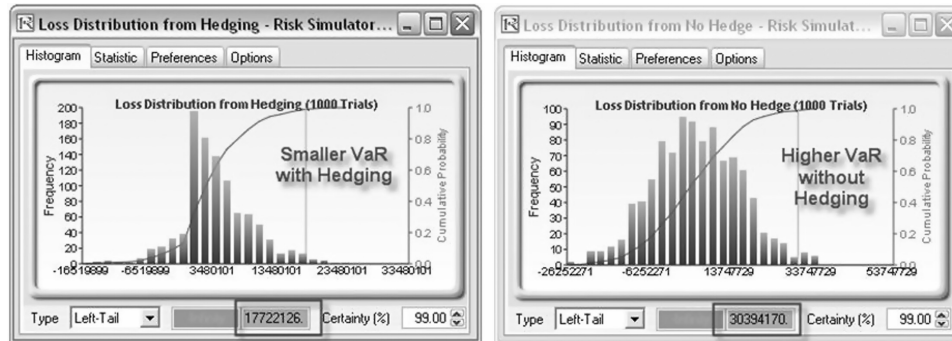
Figure 14.104 shows the effects of the Value at Risk (VaR) of a hedged versus unhedged position. Clearly the right-tailed VaR of the loss distribution is higher without the currency options hedge. Figure 14.104 shows that there is a lower risk, lower risk to returns ratio, higher returns, and less swing in the outcomes of a currency hedged position than an exposed position, with Figure 14.105 showing the simulated forecast statistics of the loss distribution. Finally, Figure 14.106 shows the hedging effectiveness, that is, how often the hedged is in the money and become usable.

**Illustrative Example: Volatility—Implied Volatility**

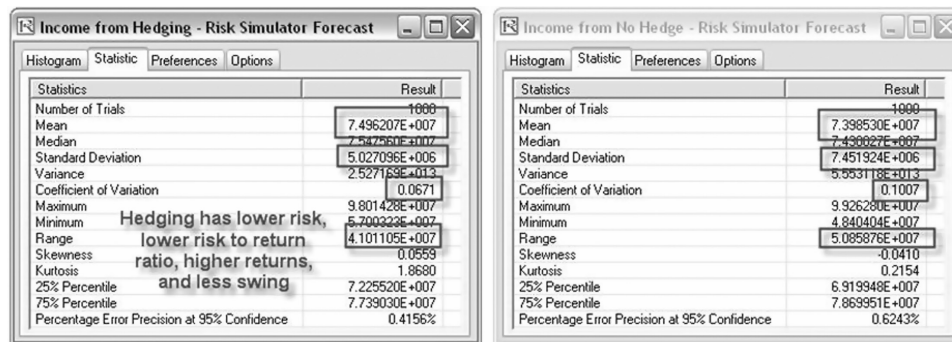
**File Name:** Volatility – Implied Volatility

**Location:** Modeling Toolkit | Volatility | Implied Volatility

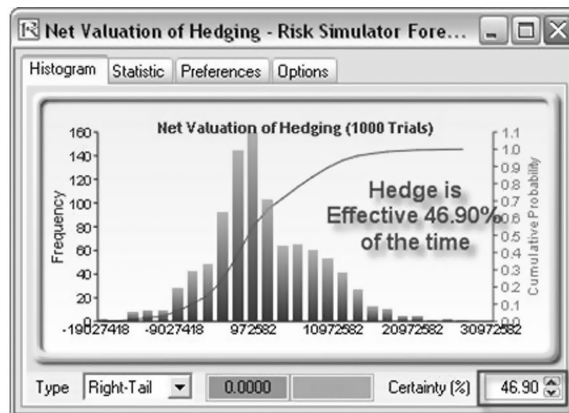
**Brief Description:** This model computes the implied volatilities using an internal optimization routine, given the values of a call or put option, as well as all their required inputs



**FIGURE 14.104** Values at Risk (VaR) of hedged versus unhedged positions.



**FIGURE 14.105** Forecast statistics of the loss distribution.



**FIGURE 14.106** Hedging effectiveness.

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Function Used:** B2ImpliedVolatilityCall, B2ImpliedVolatilityPut

This implied volatility computation is based on an internal iterative optimization, which means it will work under typical conditions (without extreme volatility values, i.e., too small or too large). It is always good modeling technique to recheck the imputed volatility using an options model to make sure the answers coincide with each other before adding more sophistication to the model. That is, given all the inputs in an option analysis as well as the option value, the volatility can be imputed (Figure 14.107).

### Illustrative Example: Volatility—Volatility Computations

**File Name:** Volatility – Volatility Computations

**Location:** *Modeling Toolkit | Volatility | Volatility Computations*

**Brief Description:** This model uses Risk Simulator to apply Monte Carlo simulation in order to compute a project's volatility measure

**Requirements:** Modeling Toolkit, Risk Simulator

There are several ways to estimate the volatility used in the option models. The most common and valid approaches are:

- **Logarithmic Cash Flow Returns Approach or Logarithmic Stock Price Returns Approach:** This method is used mainly for computing the volatility on liquid and tradable assets such as stocks in financial

### IMPLIED VOLATILITY FUNCTION

|                                |           |
|--------------------------------|-----------|
| Asset                          | \$100.00  |
| Strike                         | \$95.00   |
| Maturity                       | 0.50      |
| Risk-free                      | 8.00%     |
| Volatility                     | 25.00%    |
| DividendRate                   | 3.00%     |
|                                |           |
| Call Option                    | \$10.9126 |
| Put Option                     | \$3.6764  |
|                                |           |
| Implied Volatility Calculation |           |
| Call Option                    | 25.00%    |
| Put Option                     | 25.00%    |

**FIGURE 14.107** Getting the implied volatility from options.



options; however, it is sometimes used for other traded assets such as price of oil and price of electricity. The drawback is that discounted cash flow models with only a few cash flows will generally overstate the volatility and this method cannot be used when negative cash flows occur. This means that this volatility approach is only applicable for financial instruments and not for real options analysis. The benefits include its computational ease, transparency, and modeling flexibility of the method. In addition, no simulation is required to obtain a volatility estimate. The approach is simply to take the annualized standard deviation of the logarithmic relative returns of the time-series data as the proxy for volatility. The Modeling Toolkit function B2Volatility is used to compute this volatility, where the time series of stock prices is arranged in time series (can be chronological or reverse chronological). See the Log Cash Flow Returns example model under the Volatility section of Modeling Toolkit for details.

- **Exponentially Weighted Moving Average (EWMA) Models:** This approach is similar to the previous approach of logarithmic cash flow returns approach, using the B2Volatility function, to compute the annualized standard deviation of the natural logarithms of relative stock returns. The difference here is that the most recent value will have a higher weight than values further in the past. A *lambda* or weight variable is required (typically, industry standards set this at 0.94), where the most recent volatility is weighted at this lambda value, and the period before that is  $(1 - \text{lambda})$ , and so forth. See the EWMA example model under the Volatility section of Modeling Toolkit for details.
- **Logarithmic Present Value Returns Approach:** This approach is used mainly when computing the volatility on assets with cash flows. A typical application is in real options. The drawback of this method is that simulation is required to obtain a single volatility and is not applicable for highly traded liquid assets such as stock prices. The benefit includes the ability to accommodate certain negative cash flows and applies more rigorous analysis than the logarithmic cash flow returns approach, providing a more accurate and conservative estimate of volatility when assets are analyzed. In addition, within say, a cash flow model, multiple simulation assumptions can be set up (we can insert any types of risks and uncertainties such as related assumptions, correlated distributions and nonrelated inputs, multiple stochastic processes, and so forth), and we allow the model to distill all the interacting risks and uncertainties in these simulated assumptions, and we obtain the single value volatility, which represents the integrated risk of the project. See the Log Asset Returns example model under the Volatility section of Modeling Toolkit for details.

- **Management Assumptions and Guesses:** This approach is used for both financial options and real options. The drawback is that the volatility estimates are very unreliable and are only subjective best guesses. The benefit of this approach is its simplicity—this method is easy to explain to management the concept of volatility, both in execution and interpretation. That is, most people understand what probability is, but have a hard time understanding what volatility is. Using this approach, we can impute one from another. See the Probability to Volatility example model under the Volatility section of Modeling Toolkit for details.
- **Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Models:** These models are used mainly for computing the volatility of liquid and tradable assets such as stocks in financial options, but are sometimes used for other traded assets such as price of oil and price of electricity. The drawback is that a lot of data is required, advanced econometric modeling expertise is required, and this approach is highly susceptible to user manipulation. The benefit is that rigorous statistical analysis is performed to find the best-fitting volatility curve, providing different volatility estimates over time. The EWMA model is a simple weighting model whereas the GARCH model is a more advanced analytical and econometric model that requires advanced algorithms such as generalized method of moments to obtain the volatility forecasts.

See the GARCH model in Chapter 8 for more technical details on running this forecast approach as well as the Appendix in Chapter 2 for details on computing volatility. Risk Simulator also has the data diagnostic tool and statistical analysis tool (see Chapter 6 for details) that can quickly compute the volatility of any time-series dataset. Finally, in the Real Options SLS software folder, there is a Volatility example model that illustrates these computations in more detail.

### **Illustrative Example: Yield Curve—CIR Model**

**File Name:** Yield Curve – CIR Model

**Location:** *Modeling Toolkit | Yield Curve | CIR Model*

**Brief Description:** This is the CIR model for estimating and modeling the term structure of interest rates and yield curve approximation assuming the interest rates are mean-reverting

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Function Used:** B2CIRBondYield

The yield curve is the time-series relationship between interest rates and the time to maturity of the debt. The more formal mathematical description of this relationship is called the term structure of interest rates. The yield curve can take on various shapes. The normal yield curve means that yields rise as maturity lengthens and the yield curve is positively sloped, reflecting investor expectations for the economy to grow in the future (and hence an expectation that inflation rates will rise in the future). An inverted yield curve occurs when the opposite occurs, where the long-term yields fall below short-term yields, and long-term investors will settle for lower yields now if they think the economy will slow or even decline in the future, indicative of a worsening economic situation in the future (and hence, an expectation that inflation will remain low in the future). Another potential situation is a flat yield curve, signaling uncertainty in the economy. The yield curve can also be humped or show a smile or a frown. The yield curve over time can change in shape through a twist or bend, a parallel shift, or a movement on one end versus another.

As the yield curve is related to inflation rates as discussed above, and central banks in most countries have the ability to control monetary policy to target inflation rates, inflation rates are mean-reverting in nature. This also implies that interest rates are mean-reverting, as well as stochastically changing over time.

This section shows the Cox-Ingersoll-Ross (CIR) model that is used to compute the term structure of interest rates and yield curve (Figure 14.108). The CIR model assumes a mean-reverting stochastic interest rate. The rate of reversion and long-run mean rates can be determined using Risk Simulator's statistical analysis tool. If the long-run rate is higher than the current short rate, the yield curve is upward sloping, and vice versa.

### **Illustrative Example: Yield Curve—Curve Interpolation BIM Model**

**File Name:** Yield Curve – Curve Interpolation BIM

**Location:** *Modeling Toolkit | Yield Curve | Curve Interpolation BIM*

**Brief Description:** This is the BIM model for estimating and modeling the term structure of interest rates and yield curve approximation using a curve interpolation method

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Function Used:** B2YieldCurveBIM

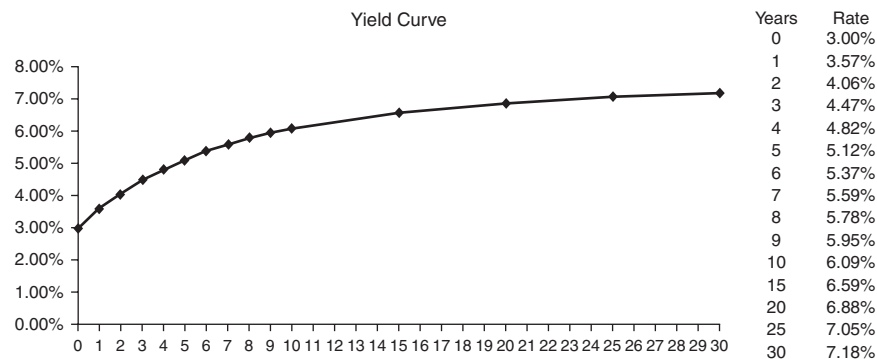
A number of alternative methods exist for estimating the term structure of interest rates and the yield curve. Some are fully specified stochastic term structure models while others are simply interpolation models. The former

### CIR MODEL YIELD CURVE CONSTRUCTION

#### Input Assumptions

|  |        |
|--|--------|
| Time to Maturity of the Bond or Debt (Years) | 1.00   |
| Risk-free Rate (Short Rate)                  | 3.00%  |
| Long-run Mean Rate                           | 8.00%  |
| Annualized Volatility of Interest Rate       | 6.00%  |
| Market Price of Interest Rate Risk           | 0.00%  |
| Rate of Mean Reversion                       | 25.00% |

**Yield of Zero Coupon Bond 3.5744%**



**FIGURE 14.108** CIR model.

are models such as the CIR and Vasicek models (illustrated in other sections in this book), while the latter are interpolation models such as the Bliss or Nelson approach. This section looks at the Bliss interpolation model (Figure 14.109) for generating the term structure of interest rates and yield curve estimation. This model requires several input parameters whereby their estimations require some econometric modeling techniques to calibrate their values. The Bliss approach is a modification of the Nelson-Siegel method by adding an additional generalized parameter. Virtually any yield curve shape can be interpolated using these models, which are widely used at banks around the world.

#### **Illustrative Example: Yield Curve—Curve Spline Interpolation and Extrapolation**

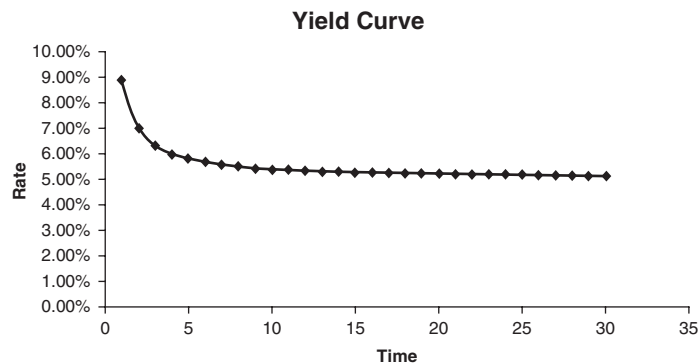
**File Name:** Yield Curve – Spline Interpolation and Extrapolation

**Location:** *Modeling Toolkit | Yield Curve | Spline Interpolation and Extrapolation*

# YIELD CURVE - INTERPOLATION MODEL

|          |        |
|----------|--------|
| Beta 0   | 0.0500 |
| Beta 1   | 0.1000 |
| Beta 2   | 0.1000 |
| Lambda 1 | 0.2000 |
| Lambda 2 | 0.2000 |

| Time | Rate  |
|------|-------|
| 0    | 8.91% |
| 1    | 7.00% |
| 2    | 6.33% |
| 3    | 6.00% |
| 4    | 5.80% |
| 5    | 5.67% |
| 6    | 5.57% |
| 7    | 5.50% |
| 8    | 5.44% |
| 9    | 5.40% |
| 10   | 5.36% |
| 11   | 5.33% |
| 12   | 5.31% |
| 13   | 5.29% |
| 14   | 5.27% |
| 15   | 5.25% |
| 16   | 5.24% |
| 17   | 5.22% |
| 18   | 5.21% |
| 19   | 5.20% |
| 20   | 5.19% |
| 21   | 5.18% |
| 22   | 5.17% |
| 23   | 5.17% |
| 24   | 5.16% |
| 25   | 5.15% |
| 26   | 5.15% |
| 27   | 5.14% |
| 28   | 5.14% |
| 29   | 5.14% |
| 30   | 5.13% |



**FIGURE 14.109** BIM Model.

**Brief Description:** This is the multidimensional cubic spline model for estimating and modeling the term structure of interest rates and yield curve approximation using a curve interpolation and extrapolation methods

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Function Used:** B2CubicSpline

The cubic spline polynomial interpolation and extrapolation model is used to “fill in the gaps” of missing spot yields and term structure of interest rates whereby the model can be used to both interpolate missing data points within a time series of interest rates (as well as other macroeconomic variables such as inflation rates and commodity prices or market returns) and also used to extrapolate outside of the given or known range, useful for forecasting purposes. In Figure 14.110, the actual U.S. Treasury risk-free rates are shown, and entered into the model as known values. The timing of these

| Years   | Spot Yields |
|---------|-------------|
| 0.0833  | 4.55%       |
| 0.2500  | 4.47%       |
| 0.5000  | 4.52%       |
| 1.0000  | 4.39%       |
| 2.0000  | 4.13%       |
| 3.0000  | 4.16%       |
| 5.0000  | 4.26%       |
| 7.0000  | 4.38%       |
| 10.0000 | 4.56%       |
| 20.0000 | 4.88%       |
| 30.0000 | 4.84%       |

These are the yields that are known and are used as inputs in the Cubic Spline Interpolation and Extrapolation model

#### Spline Interpolation and Extrapolation Results

| Years | Yield | Notes       |
|-------|-------|-------------|
| 0.5   | 4.52% | Interpolate |
| 1.0   | 4.39% | Interpolate |
| 1.5   | 4.21% | Interpolate |
| 2.0   | 4.13% | Interpolate |
| 2.5   | 4.13% | Interpolate |
| 3.0   | 4.16% | Interpolate |
| 3.5   | 4.19% | Interpolate |
| 4.0   | 4.22% | Interpolate |
| 4.5   | 4.24% | Interpolate |
| 5.0   | 4.26% | Interpolate |
| 5.5   | 4.29% | Interpolate |
| 6.0   | 4.32% | Interpolate |
| 6.5   | 4.35% | Interpolate |
| 7.0   | 4.38% | Interpolate |
| 7.5   | 4.41% | Interpolate |
| 8.0   | 4.44% | Interpolate |
| 8.5   | 4.47% | Interpolate |
| 9.0   | 4.50% | Interpolate |
| 9.5   | 4.53% | Interpolate |
| 10.0  | 4.56% | Interpolate |
| 10.5  | 4.59% | Interpolate |
| 11.0  | 4.61% | Interpolate |
| 11.5  | 4.64% | Interpolate |
| 12.0  | 4.66% | Interpolate |
| 12.5  | 4.68% | Interpolate |
| 13.0  | 4.70% | Interpolate |
| 13.5  | 4.72% | Interpolate |
| 14.0  | 4.74% | Interpolate |

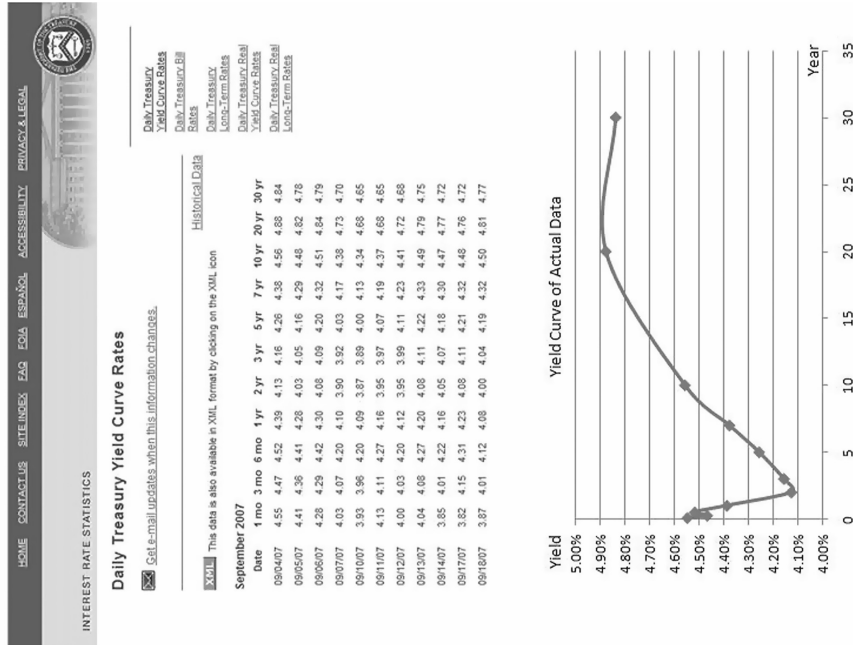


FIGURE 14.110 Cubic spline model.

spot yields are entered as Years (the known X value inputs), whereas the known risk-free rates are the known Y values. Using the “B2Cubicspline” function, we can now interpolate the in-between risk-free rates that are missing as well as the rates outside of the given input dates. For instance, the risk-free Treasury rates given include 1-month, 3-month, 6-month, 1-year, and so forth, until the 30-year rate. Using these data, we can interpolate the rates for, say, 5 months or 9 months, and so forth, as well as extrapolate beyond the 30-year rate.

### Illustrative Example: Yield Curve—Forward Rates from Spot Rates

**File Name:** Yield Curve – Forward Rates from Spot Rates

**Location:** *Modeling Toolkit | Yield Curve | Forward Rates from Spot Rates*

**Brief Description:** This is a bootstrap model used to determine the implied forward rate given two spot rates

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Function Used:** B2ForwardRate

Given two spot rates (from Year 0 to some future time periods), you can determine the implied forward rate between these two time periods. For instance, if the spot rate from Year 0 to Year 1 is 8%, and the spot rate from Year 0 to Year 2 is 7% (both yields are known currently), the implied forward rate from Year 1 to Year 2 (that will occur based on current expectations) is 6%. This is simplified by using the B2ForwardRate function in Modeling Toolkit (Figure 14.111).

#### FORWARD RATES COMPUTING FORWARD RATES FROM SPOT RATES

##### Input Assumptions

|                     |       |
|---------------------|-------|
| Spot Rate 1         | 8.00% |
| Spot Rate 2         | 7.00% |
| Time of Spot Rate 1 | 1.00  |
| Time of Spot Rate 2 | 2.00  |

**Forward Rate** **6.00%**

**FIGURE 14.111** Forward rate extrapolation.

### Illustrative Example: Yield Curve—Vasicek Model

**File Name:** Yield Curve – Vasicek Model

**Location:** *Modeling Toolkit | Yield Curve | Vasicek Model*

**Brief Description:** The Vasicek model is used to create the term structure of interest rates and to reconstruct the yield curve assuming the underlying interest rates are mean-reverting and stochastic

**Requirements:** Modeling Toolkit, Risk Simulator

**Modeling Toolkit Function Used:** B2VasicekBondYield

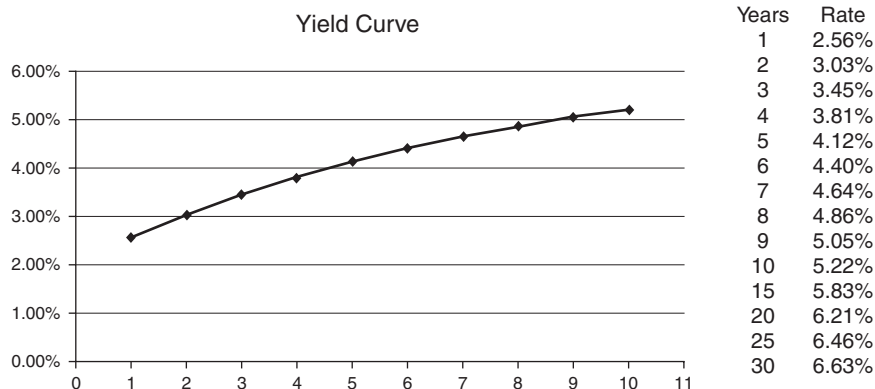
This is the Vasicek model used to compute the term structure of interest rates and yield curve. The Vasicek model assumes a mean-reverting stochastic interest rate (Figure 14.112). The rate of reversion and long-run mean rates can be determined using Risk Simulator's statistical analysis tool. If the

#### VASICEK MODEL YIELD CURVE CONSTRUCTION

##### Input Assumptions

|  |        |
|--|--------|
| Time to Maturity of the Bond or Debt (Years) | 1.00   |
| Risk-free Rate (Short Rate)                  | 2.00%  |
| Long-run Mean Rate                           | 8.00%  |
| Annualized Volatility of Interest Rate       | 2.00%  |
| Market Price of Interest Rate Risk           | 0.00%  |
| Rate of Mean Reversion                       | 20.00% |

**Yield of Zero Coupon Bond** **2.5562%**



**FIGURE 14.112** Using the Vasicek model to generate a yield curve.



long-run rate is higher than the current short rate, the yield curve is upward sloping, and vice versa.

The yield curve is the time-series relationship between interest rates and the time to maturity of the debt. The more formal mathematical description of this relationship is called the term structure of interest rates. As discussed previously, the yield curve can take on various shapes. The normal yield curve means that yields rise as maturity lengthens and the yield curve is positively sloped, reflecting investor expectations for the economy to grow in the future (and hence an expectation that inflation rates will rise in the future). An inverted yield curve occurs when the opposite occurs, where the long-term yields fall below short-term yields, and long-term investors will settle for lower yields now if they think the economy will slow or even decline in the future, indicative of a worsening economic situation in the future (and hence an expectation that inflation will remain low in the future). Another potential situation is a flat yield curve, signaling uncertainty in the economy. The yield curve can also be humped or show a smile or a frown. The yield curve over time can change in shape through a twist or bend, a parallel shift, or a movement on one end versus another.

As the yield curve is related to inflation rates as discussed above, and central banks in most countries have the ability to control monetary policy to target inflation rates, inflation rates are mean-reverting in nature. This also implies that interest rates are mean-reverting, as well as stochastically changing over time.

A Czech mathematician, Oldrich Vasicek, in a 1977 paper proved that bond prices on a yield curve over time and various maturities are driven by the short end of the yield curve, or the short-term interest rates, using a risk-neutral martingale measure. In his work the mean-reverting Ornstein-Uhlenbeck process was assumed, hence the resulting Vasicek model requires that a mean-reverting interest rate process be modeled (rate of mean reversion and long-run mean rates are both inputs in the Vasicek model).

### **Illustrative Example: Stochastic Forecasting of Interest Rates and Stock Prices**

**File Name:** Forecasting – Stochastic Processes

**Location:** *Modeling Toolkit | Forecasting | Stochastic Processes*

**Brief Description:** This sample model illustrates how to simulate Stochastic Processes (Brownian Motion Random Walk, Mean-Reversion, Jump-Diffusion, and Mixed Models)

**Requirements:** Modeling Toolkit, Risk Simulator

A stochastic process is a sequence of events or paths generated by probabilistic laws. That is, random events can occur over time but are governed by specific statistical and probabilistic rules. The main stochastic processes include Random Walk or Brownian Motion, Mean-Reversion, and Jump-Diffusion. These processes can be used to forecast a multitude of variables that seemingly follow random trends but yet are restricted by probabilistic laws. We can use Risk Simulator's *Stochastic Process* module to simulate and create such processes. These processes can be used to forecast a multitude of time-series data including stock prices, interest rates, inflation rates, oil prices, electricity prices, commodity prices, and so forth.

### Stochastic Process Forecasting

To run this model, simply:

1. Select *Simulation | Forecasting | Stochastic Processes*.
2. Enter a set of relevant inputs or use the existing inputs as a test case (Figure 14.113).

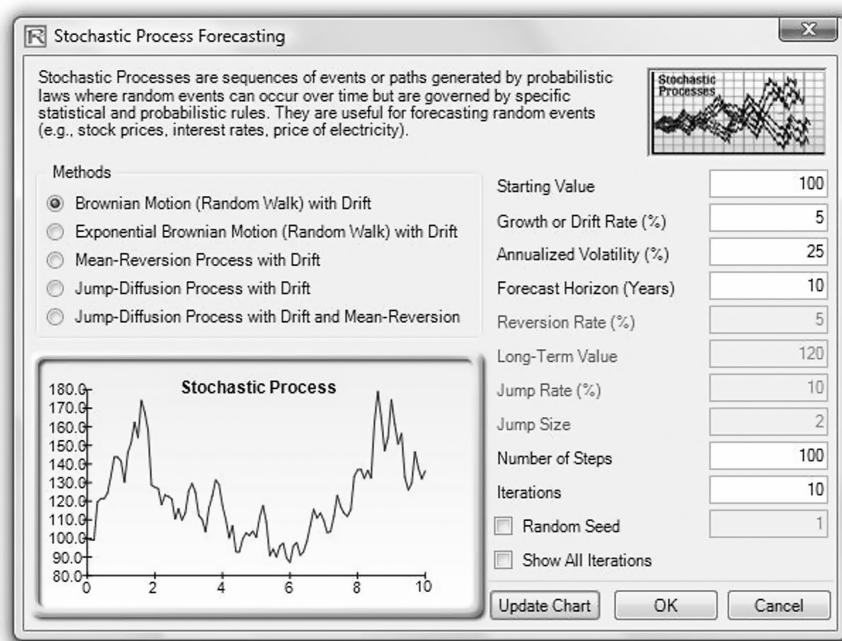


FIGURE 14.113 Running a stochastic process forecast.

3. Select the relevant process to simulate.
4. Click on *Update Chart* to view the updated computation of a single path or click *OK* to create the process.

### **Model Results Analysis**

For your convenience, the analysis report sheet is included in the model. A stochastic time-series chart and forecast values are provided in the report as well as each step's time period, mean, and standard deviation of the forecast (see Chapter 8 for technical details of using this stochastic process forecast technique and Chapter 6 for the data diagnostic and statistical analysis tools for calibrating the input parameters for the stochastic process models). The mean values can be used as the single point estimate or assumptions can be manually generated for the desired time period. That is, finding the appropriate time period, create an assumption with a normal distribution with the appropriate mean and standard deviation computed. A sample chart with 10 iteration paths is included to graphically illustrate the behavior of the forecasted process.

### **CASE STUDY: IT INFORMATION SECURITY INTRUSION RISK MANAGEMENT**

*This case study illustrates an information systems security attack profile and provides decision analysis and support on the optimal investment. The model is contributed by Mark A. Benyovszky, Managing Director of Zero Delta Center for Enterprise Alignment. Zero Delta CfEA is a research and development organization that specializes in helping companies to align their strategic and tactical efforts.*

There are several models illustrated in this case study and available in the Modeling Toolkit. First, the IT Bandwidth Requirements for Streaming Media model shows how to create forecasts for media streaming environments for off-peak and peak media consumption cycles. The model explains how to use historical time-series data to establish an understanding of the future demand for media consumption. This model also explores how the Delphi Method can be used when historical data do not exist or when there is significant uncertainty surrounding the demand for new media types that require more bandwidth or have chattier communication channels.

The IT Intrusion Management model shows how to create an information systems security attack profile, determine the probabilities of occurrence of different types of attacks, assess the financial and operational impact that an attack has on an organization, and arrive at the level of investment in