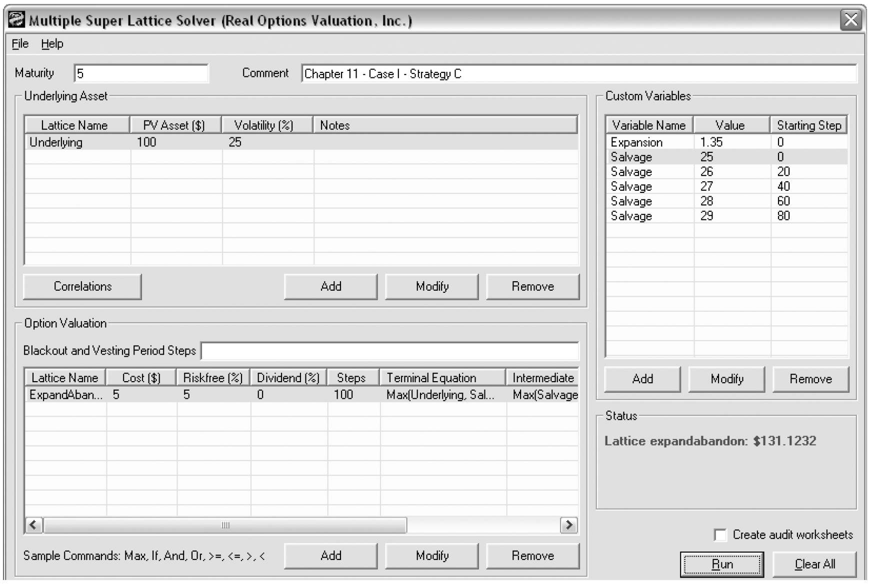

FIGURE 11.6 Value of Strategy C

$55.22M for the start-up (i.e., $50M + $81.12M – $75.90M), otherwise it is better off pursuing Strategy B and building the technology itself.

Thus, the optimal strategy is to purchase the start-up company, go to market quickly with the ability to abandon and sell the start-up should things fail, or to further invest an additional R&D sum later on to develop spin-off technologies. If real options analysis was not performed, Microtech would have chosen to develop the technology itself immediately and spend $40M. This strategy would yield the highest NPV if real options and risk mitigation options are not considered. Microtech would have made a serious decision blunder and taken unnecessary risks. By performing the real options analysis, additional spin-off products and opportunities surface, which prove to be highly valuable.

CASE 2: FINANCIAL OPTIONS—CONVERTIBLE WARRANTS WITH A VESTING PERIOD AND PUT PROTECTION

This case study provides a sample application of the Super Lattice Solver on valuing a warrant (an instrument that can be converted into a stock, similar to a call option) that has a protective put option associated with it. The analysis herein also applies both a customized binomial lattice and a closed-form Black-Scholes model for comparison and benchmarking purposes.

The valuation analysis performed was based on an actual consulting project but all proprietary information provided by the client has been modified except for certain basic publicly available information, and the accuracy of said results is dependent on this factual information at the time of valuation. This case details the input assumptions used as well as some benchmark due diligence to check the results. Certain technical details have been left out to simplify the case.

The client company very recently acquired a small IT firm. The acquisition consisted of both cash as well as some warrants, which are convertible into stocks. But because the client's stocks are fairly volatile, the acquired firm negotiated a protective put to hedge its downside risks. In return for providing this protective put, the client requested that the warrant be exercisable only if the target firm is solvent and its gross margins exceed 33 percent and be no less than $10 million.

Clearly, this problem cannot be solved using a Black-Scholes model because there exist dividends, a vesting period, a threshold price put protection at which the put can be exercised, and the fact that the put cannot be exercised unless the warrant is converted into a stock but only when the stock price is below $33.

To summarize, the following list shows the assumptions and requirements in this exotic warrant:

Stock price on grant date:	$30.12
Warrant strike price:	$15.00
Warrant maturity:	10.00 years (grant date)
Risk-free rate:	4.24% (grant date)
Volatility:	29.50%
Dividend rate:	0.51%
Put threshold price:	$33.00
Vesting for warrant:	3 years
Vesting for put option:	5 years

Further, the following requirements were modeled:

- The protective put option can only be exercised if the warrant is exercised.
- The put option can only be exercised if the stock price is below $33.00 at the time of exercise.
- The warrant can only be exercised if recipient's gross margin equals or exceeds 33 percent and be no less than $10 million. A simulation forecast puts an 85 percent to 90 percent uniform probability of occurrence for this event.

- The warrant can only be exercised if recipient is solvent. Another simulation forecast puts a 90 percent to 95 percent uniform probability of occurrence for this event.
- The protective put payout is the maximum spread between the put threshold price less client's common stock price or the warrant price.

The risk-free rate is based on the 10-year U.S. Treasury note. The volatility estimate is based on the client's historical weekly closing prices for the past one, two, and three years. The volatilities are estimated using the standard deviation of the natural logarithmic relative returns, annualized for a year, and then averaged. The dividend rate is assumed to be 0.51 percent based on available market data on client shares. The total probability of exceeding the gross margin threshold as well as solvency requirements is 80.9375 percent (calculated using the midpoint probability estimates of both independent events 87.50 percent times 92.50 percent, and were results based on simulation forecasts using historical financial data). The only method applicable in valuing such a protective put on a warrant is the use of binomial lattices. However, a Black-Scholes model is used to benchmark the results.

Warrant Valuation

In order to solve the warrant part of the exotic vehicle, high-level analysis rules need to be created:

- If the period ≥ 3 years, then at maturity, the profit maximizing decision is: Max (Exercise the warrant accounting for the probability the requirements are met; or let the warrant expire worthless otherwise).
- If the period ≥ 3 years, then prior to maturity, the profit maximizing decision is: Max (Exercise the warrant accounting for the probability the requirements are met; or keep the option open for future execution).
- If the period < 3 years, then hold on to the warrant as no execution is allowed.

Protective Put Option Valuation

The same is done on the protective put option:

- If the period ≥ 5 years, then at maturity, the profit maximizing decision is: Max (If the stock price is < $33, then exercise the warrant and collect the protective put payout, after accounting for the probability the requirements are met; or let the warrant and put option expire worthless).
- If the period ≥ 5 years, then prior to maturity, the profit maximizing decision is: Max (If the stock price is < $33, then exercise the warrant and

collect the protective put payout, after accounting for the probability the requirements are met; or keep the option open for future execution).

- If the period < 5 years, then hold on to the put option as no execution is allowed.

The binomial model used is a combination of Bermudan vesting nested option, where all the requirements (vesting periods, threshold price, probability of solvency, probability of exceeding gross margin requirements) have to be met before the warrant can be executed, and the put option can only be executed if the warrant is executed. However, the warrant can be executed even if the protective put is not executed.

Analytical Results

A summary of the results of the analysis follows. The results start with a decomposition of the warrant call and the protective put valued independently. These results are then compared to benchmarks to ascertain their accuracy and model reliability. Then, a combination of both instruments is valued in a mutually exclusive nested option model. The results of interest are the combined option model, but we can only obtain such a model by first decomposing the parts. The analysis was performed using the Super Lattice Solver software.

To follow along, you can start the Single Asset Super Lattice Solver software and load the relevant example files: *Case Study - Warrant - Warrant Only; Case Study - Warrant - Put Only*; and *Case Study - Warrant - Combined Value.sls*.

A. Warrant at Grant Date

Naïve Black-Scholes (benchmark)	$19.71
Adjusted Black-Scholes (benchmark)	$15.95 (probability adjusted benchmark)
Binomial lattice (100 steps)	$15.98 (using Super Lattice Solver)

As can be seen, the binomial lattice for the warrant converges to the Black-Scholes results. The reason for this convergence is that the dividend rate is low, making it not optimal to exercise early, but still worth slightly more than a simple European option. See Figure 11.7 for the details.

B. Protective Put Option at Grant Date

Static protection value (total)	$1.5 million (100,000 warrants granted)
Static protection value (per warrant)	$15.00 (guaranteed minimum)

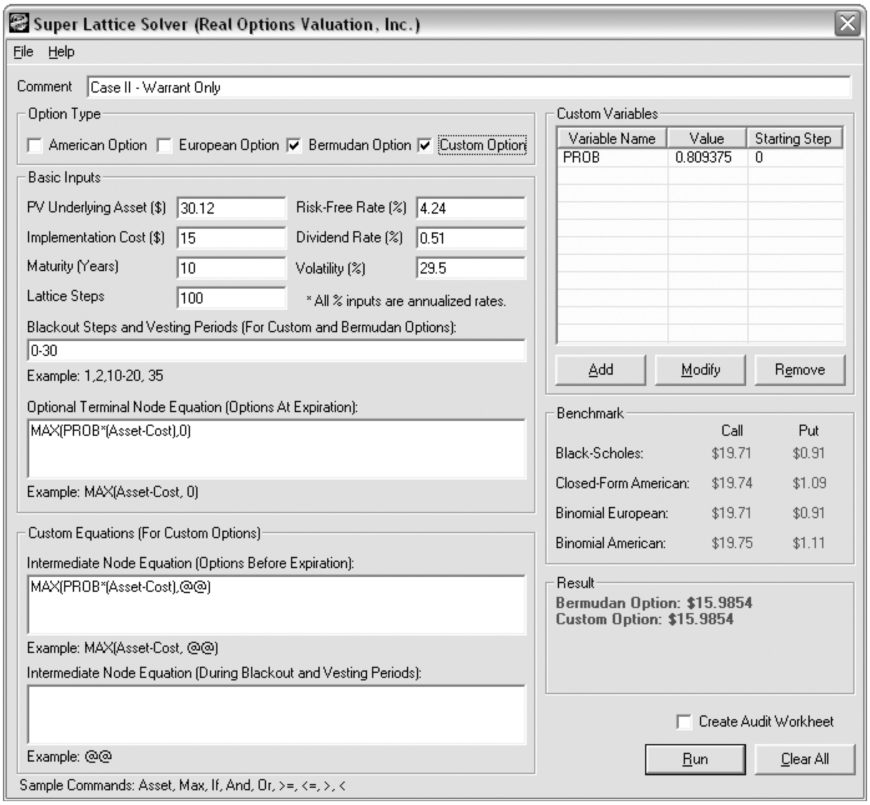

FIGURE 11.7 Warrant Valuation at Grant Date

Adjusted static protection value	\$12.14 (probability adjusted benchmark)
Binomial lattice (100 steps)	\$12.08 (using Super Lattice Solver)

The analysis can be seen in Figure 11.8.

The analysis up to this point decomposes the warrant call and the protective put options and their values are comparable to the static benchmarks, indicating that the models are correctly specified and the results are accurate. However, the warrant issues cannot be separated from the protective put because they are combined into one instrument. Separating them means that at certain points and conditions in time, the holder can both execute the call and also execute the put option protection with another call. This constitutes double-counting. Thus, in such a mutually exclusive condition (either a call is executed or a protective put is executed with the call, not both), a combination

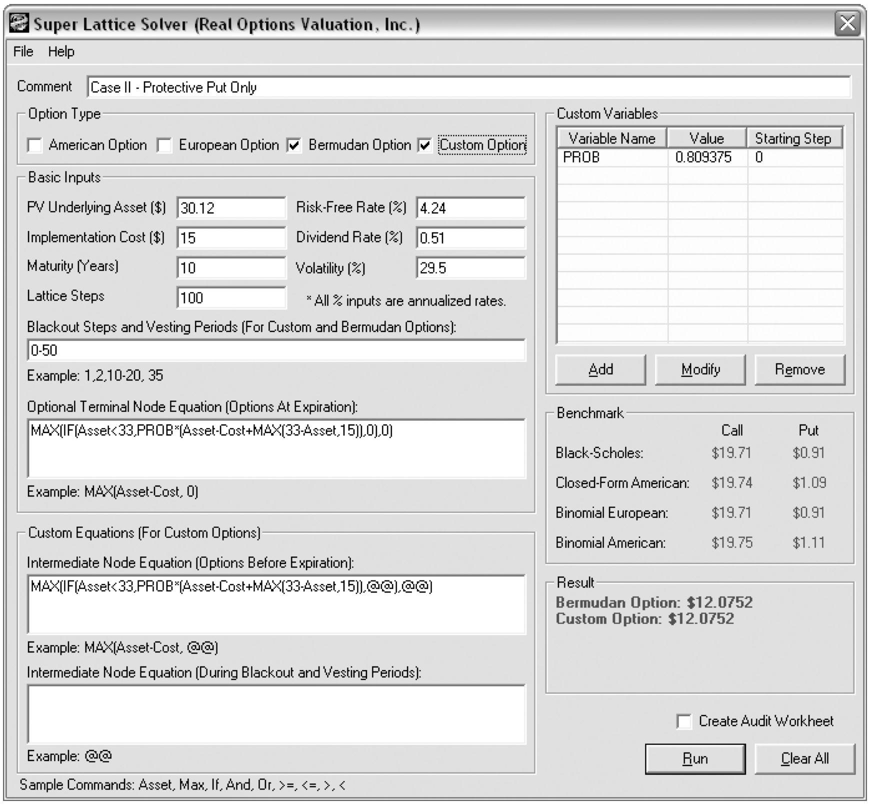

FIGURE 11.8 Protective Put at Grant Date

valuation is performed and the results are shown in the following list. Figure 11.9 illustrates the analysis performed.

C. Combination of Warrant and Protective Put Option at Grant Date

Black-Scholes call option	$19.71 (benchmark)
Black-Scholes put option	$ 0.91 (benchmark)
Combination of both Black-Scholes	$20.62 (sum of both benchmarks)
Binomial lattice (100 steps)	$22.37

Using Black-Scholes call and put option models as benchmarks, we see that the sought-after result of $22.37 is valid, after considering that the decompositions of the model are also valid. Clearly the total combination value has to exceed the Black-Scholes as the warrant-put is an American op-

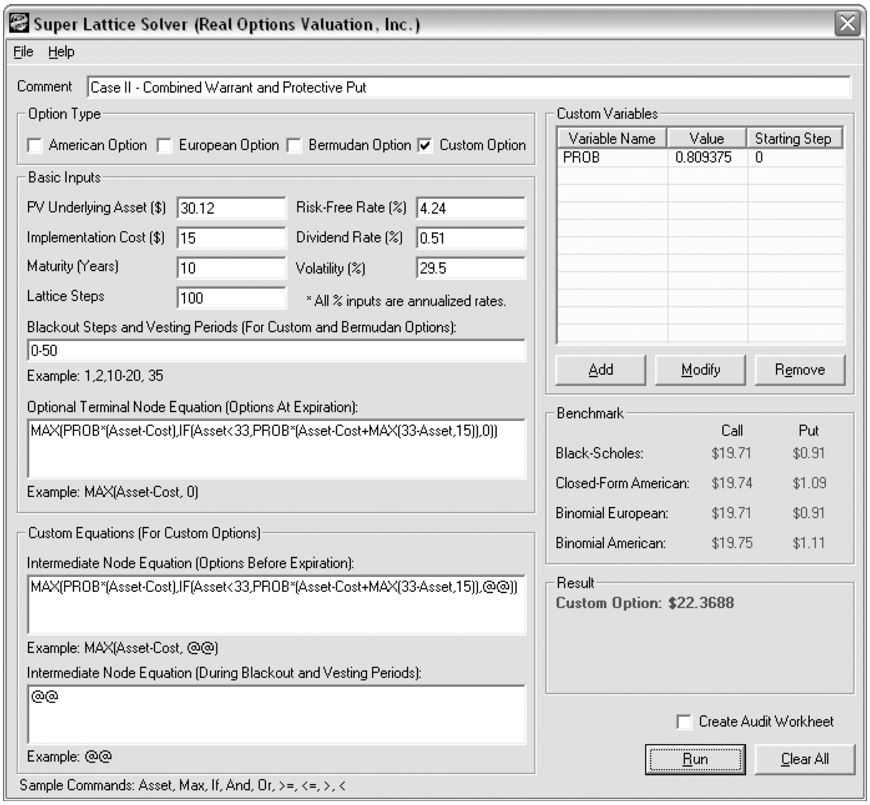

FIGURE 11.9 Combined Warrant with Protective Put at Grant Date

tion (with vesting requirements). To summarize, the analysis cannot be completed without the use of the Single Asset SLS software, and even when solving such complicated instruments, the pricing is relatively straightforward when using the software.

CASE 3: PHARMACEUTICAL DEVELOPMENT—VALUE OF PERFECT INFORMATION AND OPTIMAL TRIGGER VALUES

Suppose BioGen, a large multibillion dollar pharmaceutical firm is thinking of developing a new type of insulin that can be inhaled and the drug will directly be absorbed into the blood stream. This is indeed a novel and honorable idea. Imagine what this means to diabetics who no longer need painful and frequent