Risky ventures are the norm in the daily business world. The mere mention of names such as George Soros, John Meriweather, Paul Reichmann, and Nicholas Leeson, or firms such as Long Term Capital Management, Metallgesellschaft, Barings Bank, Bankers Trust, Daiwa Bank, Sumimoto Corporation, Merrill Lynch, and Citibank brings a shrug of disbelief and fear. These names are some of the biggest in the world of business and finance. Their claim to fame is not simply being the best and brightest individuals or being the largest and most respected firms, but for bearing the stigma of being involved in highly risky ventures that turned sour almost overnight.1

George Soros was and still is one of the most respected names in high finance; he is known globally for his brilliance and exploits. Paul Reichmann was a reputable and brilliant real estate and property tycoon. Between the two of them, nothing was impossible, but when they ventured into investments in Mexican real estate, the wild fluctuations of the peso in the foreign exchange market was nothing short of a disaster. During late 1994 and early 1995, the peso hit an all-time low and their ventures went from bad to worse, but the one thing that they did not expect was that the situation would become a lot worse before it was all over and billions would be lost as a consequence.

Long Term Capital Management was headed by Meriweather, one of the rising stars in Wall Street, with a slew of superstars on its management team, including several Nobel laureates in finance and economics (Robert Merton and Myron Scholes). The firm was also backed by giant investment banks. A firm that seemed indestructible literally blew up with billions of dollars in the red, shaking the international investment community with repercussions throughout Wall Street as individual investors started to lose faith in large hedge funds and wealth-management firms, forcing the eventual massive Federal Reserve bailout.

Barings was one of the oldest banks in England. It was so respected that even Queen Elizabeth II herself held a private account with it. This multibillion dollar institution was brought down single-handedly by Nicholas Leeson, an employee halfway around the world. Leeson was a young and
brilliant investment banker who headed up Barings' Singapore branch. His illegally doctored track record showed significant investment profits, which gave him more leeway and trust from the home office over time. He was able to cover his losses through fancy accounting and by taking significant amounts of risk. His speculations in the Japanese yen went south and he took Barings down with him, and the top echelon in London never knew what hit them.

Had any of the managers in the boardroom at their respective headquarters bothered to look at the risk profile of their investments, they would surely have made a very different decision much earlier on, preventing what became major embarrassments in the global investment community. If the projected returns are adjusted for risks, that is, finding what levels of risks are required to attain such seemingly extravagant returns, it would be sensible not to proceed.

Risks occur in everyday life that do not require investments in the multimillions. For instance, when would one purchase a house in a fluctuating housing market? When would it be more profitable to lock in a fixed-rate mortgage rather than keep a floating variable rate? What are the chances that there will be insufficient funds at retirement? What about the potential personal property losses when a hurricane hits? How much accident insurance is considered sufficient? How much is a lottery ticket actually worth?

Risk permeates all aspects of life and one can never avoid taking or facing risks. What we can do is understand risks better through a systematic assessment of their impacts and repercussions. This assessment framework must also be capable of measuring, monitoring, and managing risks; otherwise, simply noting that risks exist and moving on is not optimal. This book provides the tools and framework necessary to tackle risks head-on. Only with the added insights gained through a rigorous assessment of risk can one actively manage and monitor risk.

Risks permeate every aspect of business, but we do not have to be passive participants. What we can do is develop a framework to better understand risks through a systematic assessment of their impacts and repercussions. This framework also must be capable of measuring, monitoring, and managing risks.

THE BASICS OF RISK

Risk can be defined simply as any uncertainty that affects a system in an unknown fashion whereby the ramifications are also unknown but bears with
it great fluctuation in value and outcome. In every instance, for risk to be evident, the following generalities must exist:

- Uncertainties and risks have a time horizon.
- Uncertainties exist in the future and will evolve over time.
- Uncertainties become risks if they affect the outcomes and scenarios of the system.
- These changing scenarios’ effects on the system can be measured.
- The measurement has to be set against a benchmark.

Risk is never instantaneous. It has a time horizon. For instance, a firm engaged in a risky research and development venture will face significant amounts of risk but only until the product is fully developed or has proven itself in the market. These risks are caused by uncertainties in the technology of the product under research, uncertainties about the potential market, uncertainties about the level of competitive threats and substitutes, and so forth. These uncertainties will change over the course of the company’s research and marketing activities—some uncertainties will increase while others will most likely decrease through the passage of time, actions, and events. However, only the uncertainties that affect the product directly will have any bearing on the risks of the product being successful. That is, only uncertainties that change the possible scenario outcomes will make the product risky (e.g., market and economic conditions). Finally, risk exists if it can be measured and compared against a benchmark. If no benchmark exists, then perhaps the conditions just described are the norm for research and development activities, and thus the negative results are to be expected. These benchmarks have to be measurable and tangible, for example, gross profits, success rates, market share, time to implementation, and so forth.
looks at the nature of risk and return. Markowitz did not look at risk as the enemy but as a condition that should be embraced and balanced out through its expected returns. The concept of risk and return was then refined through later works by William Sharpe and others, who stated that a heightened risk necessitates a higher return, as elegantly expressed through the capital asset pricing model (CAPM), where the required rate of return on a marketable risky equity is equivalent to the return on an equivalent riskless asset plus a beta systematic and undiversifiable risk measure multiplied by the market risk’s return premium. In essence, a higher risk asset requires a higher return. In Markowitz’s model, one could strike a balance between risk and return. Depending on the risk appetite of an investor, the optimal or best-case returns can be obtained through the efficient frontier. Should the investor require a higher level of returns, he or she would have to face a higher level of risk. Markowitz’s work carried over to finding combinations of individual projects or assets in a portfolio that would provide the best bang for the buck, striking an elegant balance between risk and return. In order to better understand this balance, also known as risk adjustment in modern risk analysis language, risks must first be measured and understood. The following section illustrates how risk can be measured.

THE STATISTICS OF RISK

The study of statistics refers to the collection, presentation, analysis, and utilization of numerical data to infer and make decisions in the face of uncertainty, where the actual population data is unknown. There are two branches in the study of statistics: descriptive statistics, where data is summarized and described, and inferential statistics, where the population is generalized through a small random sample, such that the sample becomes useful for making predictions or decisions when the population characteristics are unknown.

A sample can be defined as a subset of the population being measured, whereas the population can be defined as all possible observations of interest of a variable. For instance, if one is interested in the voting practices of all U.S. registered voters, the entire pool of a hundred million registered voters is considered the population, whereas a small survey of one thousand registered voters taken from several small towns across the nation is the sample. The calculated characteristics of the sample (e.g., mean, median, standard deviation) are termed statistics, while parameters imply that the entire population has been surveyed and the results tabulated. Thus, in decision making, the statistic is of vital importance, seeing that sometimes the entire population is yet unknown (e.g., who are all your customers, what is the total market share, etc.) or it is very difficult to obtain all relevant
information on the population seeing that it would be too time- or resource-
consuming.

In inferential statistics, the usual steps undertaken include:

■ Designing the experiment—this phase includes designing the ways to collect all possible and relevant data.
■ Collection of sample data—data is gathered and tabulated.
■ Analysis of data—statistical analysis is performed.
■ Estimation or prediction—inferences are made based on the statistics obtained.
■ Hypothesis testing—decisions are tested against the data to see the outcomes.
■ Goodness-of-fit—actual data is compared to historical data to see how accurate, valid, and reliable the inference is.
■ Decision making—decisions are made based on the outcome of the inference.

**Measuring the Center of the Distribution—The First Moment**

The first moment of a distribution measures the *expected rate of return* on a particular project. It measures the location of the project’s scenarios and possible outcomes on average. The common statistics for the first moment include the mean (average), median (center of a distribution), and mode (most commonly occurring value). Figure 2.1 illustrates the first moment—where, in this case, the first moment of this distribution is measured by the mean ($\mu$) or average value.

**Measuring the Spread of the Distribution—The Second Moment**

The second moment measures the spread of a distribution, which is a *measure of risk*. The spread or width of a distribution measures the variability of
a variable, that is, the potential that the variable can fall into different regions of the distribution—in other words, the potential scenarios of outcomes. Figure 2.2 illustrates two distributions with identical first moments (identical means) but very different second moments or risks. The visualization becomes clearer in Figure 2.3. As an example, suppose there are two stocks and the first stock’s movements (illustrated by the darker line) with the smaller fluctuation is compared against the second stock’s movements (illustrated by the dotted line) with a much higher price fluctuation. Clearly an investor would view the stock with the wilder fluctuation as riskier because the outcomes of the more risky stock are relatively more unknown.
than the less risky stock. The vertical axis in Figure 2.3 measures the stock prices; thus, the more risky stock has a wider range of potential outcomes. This range is translated into a distribution’s width (the horizontal axis) in Figure 2.2, where the wider distribution represents the riskier asset. Hence, width or spread of a distribution measures a variable’s risks.

Notice that in Figure 2.2, both distributions have identical first moments or central tendencies, but clearly the distributions are very different. This difference in the distributional width is measurable. Mathematically and statistically, the width or risk of a variable can be measured through several different statistics, including the range, standard deviation ($\sigma$), variance, coefficient of variation, volatility, and percentiles.

**Measuring the Skew of the Distribution—The Third Moment**

The third moment measures a distribution’s skewness, that is, how the distribution is pulled to one side or the other. Figure 2.4 illustrates a negative or left skew (the tail of the distribution points to the left) and Figure 2.5 illustrates a positive or right skew (the tail of the distribution points to the right). The mean is always skewed toward the tail of the distribution while the median remains constant. Another way of seeing this is that the mean

![Skew distribution](image)

**FIGURE 2.4** Third moment (left skew).

![Skew distribution](image)

**FIGURE 2.5** Third moment (right skew).
moves, but the standard deviation, variance, or width may still remain constant. If the third moment is not considered, then looking only at the expected returns (e.g., mean or median) and risk (standard deviation), a positively skewed project might be incorrectly chosen! For example, if the horizontal axis represents the net revenues of a project, then clearly a left or negatively skewed distribution might be preferred as there is a higher probability of greater returns (Figure 2.4) as compared to a higher probability for lower level returns (Figure 2.5). Thus, in a skewed distribution, the median is a better measure of returns, as the medians for both Figures 2.4 and 2.5 are identical, risks are identical, and, hence, a project with a negatively skewed distribution of net profits is a better choice. Failure to account for a project’s distributional skewness may mean that the incorrect project may be chosen (e.g., two projects may have identical first and second moments, that is, they both have identical returns and risk profiles, but their distributional skews may be very different).

**Measuring the Catastrophic Tail Events of the Distribution—The Fourth Moment**

The fourth moment, or kurtosis, measures the peakedness of a distribution. Figure 2.6 illustrates this effect. The background (denoted by the dotted line) is a normal distribution with an excess kurtosis of 0. The new distribution has a higher kurtosis; thus the area under the curve is thicker at the tails with less area in the central body. This condition has major impacts on risk analysis as for the two distributions in Figure 2.6; the first three moments (mean, standard deviation, and skewness) can be identical, but the fourth moment (kurtosis) is different. This condition means that, although the returns and risks are identical, the probabilities of extreme and catastrophic
events (potential large losses or large gains) occurring are higher for a high kurtosis distribution (e.g., stock market returns are leptokurtic or have high kurtosis). Ignoring a project’s return’s kurtosis may be detrimental. Note that sometimes a normal kurtosis is denoted as 3.0, but in this book we use the measure of excess kurtosis, henceforth simply known as kurtosis. In other words, a kurtosis of 3.5 is also known as an excess kurtosis of 0.5, indicating that the distribution has 0.5 additional kurtosis above the normal distribution. The use of excess kurtosis is more prevalent in academic literature and is, hence, used here. Finally, the normalization of kurtosis to a base of 0 makes for easier interpretation of the statistic (e.g., a positive kurtosis indicates fatter-tailed distributions while negative kurtosis indicates thinner-tailed distributions).

Most distributions can be defined up to four moments. The first moment describes the distribution’s location or central tendency (expected returns), the second moment describes its width or spread (risks), the third moment its directional skew (most probable events), and the fourth moment its peakedness or thickness in the tails (catastrophic losses or gains). All four moments should be calculated and interpreted to provide a more comprehensive view of the project under analysis.

**THE MEASUREMENTS OF RISK**

There are multiple ways to measure risk in projects. This section summarizes some of the more common measures of risk and lists their potential benefits and pitfalls. The measures include:

- **Probability of Occurrence.** This approach is simplistic and yet effective. As an example, there is a 10 percent probability that a project will not break even (it will return a negative net present value indicating losses) within the next 5 years. Further, suppose two similar projects have identical implementation costs and expected returns. Based on a single-point estimate, management should be indifferent between them. However, if risk analysis such as Monte Carlo simulation is performed, the first project might reveal a 70 percent probability of losses compared to only a 5 percent probability of losses on the second project. Clearly, the second project is better when risks are analyzed.

- **Standard Deviation and Variance.** Standard deviation is a measure of the average of each data point’s deviation from the mean. This is the
most popular measure of risk, where a higher standard deviation implies a wider distributional width and, thus, carries a higher risk. The drawback of this measure is that both the upside and downside variations are included in the computation of the standard deviation. Some analysts define risks as the potential losses or downside; thus, standard deviation and variance will penalize upswings as well as downsides.

- **Semi-Standard Deviation.** The semi-standard deviation only measures the standard deviation of the downside risks and ignores the upside fluctuations. Modifications of the semi-standard deviation include calculating only the values below the mean, or values below a threshold (e.g., negative profits or negative cash flows). This provides a better picture of downside risk but is more difficult to estimate.

- **Volatility.** The concept of volatility is widely used in the applications of real options and can be defined briefly as a measure of uncertainty and risks. Volatility can be estimated using multiple methods, including simulation of the uncertain variables impacting a particular project and estimating the standard deviation of the resulting asset’s logarithmic returns over time. This concept is more difficult to define and estimate but is more powerful than most other risk measures in that this single value incorporates all sources of uncertainty rolled into one value.

- **Beta.** Beta is another common measure of risk in the investment finance arena. Beta can be defined simply as the undiversifiable, systematic risk of a financial asset. This concept is made famous through the CAPM, where a higher beta means a higher risk, which in turn requires a higher expected return on the asset.

- **Coefficient of Variation.** The coefficient of variation is simply defined as the ratio of standard deviation to the mean, which means that the risks are common-sized. For example, the distribution of a group of students’ heights (measured in meters) can be compared to the distribution of the students’ weights (measured in kilograms). This measure of risk or dispersion is applicable when the variables’ estimates, measures, magnitudes, or units differ.

- **Value at Risk.** Value at Risk (VaR) was made famous by J. P. Morgan in the mid-1990s through the introduction of its RiskMetrics approach, and has thus far been sanctioned by several bank governing bodies around the world. Briefly, it measures the amount of capital reserves at risk given a particular holding period at a particular probability of loss. This measurement can be modified to risk applications by stating, for example, the amount of potential losses a certain percent of the time during the period of the economic life of the project—clearly, a project with a smaller VaR is better.

- **Worst-Case Scenario and Regret.** Another simple measure is the value of the worst-case scenario given catastrophic losses. Another definition is
regret. That is, if a decision is made to pursue a particular project, but if the project becomes unprofitable and suffers a loss, the level of regret is simply the difference between the actual losses compared to doing nothing at all.

- Risk-Adjusted Return on Capital. Risk-adjusted return on capital (RAROC) takes the ratio of the difference between the fiftieth percentile (median) return and the fifth percentile return on a project to its standard deviation. This approach is used mostly by banks to estimate returns subject to their risks by measuring only the potential downside effects and ignoring the positive upswings.

The following appendix details the computations of some of these risk measures and is worthy of review before proceeding through the book.

**APPENDIX—COMPUTING RISK**

This appendix illustrates how some of the more common measures of risk are computed. Each risk measurement has its own computations and uses. For example, certain risk measures are applicable only on time-series data (e.g., volatility) while others are applicable in both cross-sectional and time-series data (e.g., variance, standard deviation, and covariance), while others require a consistent holding period (e.g., Value at Risk) or a market comparable or benchmark (e.g., beta coefficient).

**Probability of Occurrence**

This approach is simplistic yet effective. The probability of success or failure can be determined several ways. The first is through management expectations and assumptions, also known as expert opinion, based on historical occurrences or experience of the expert. Another approach is simply to gather available historical or comparable data, industry averages, academic research, or other third-party sources, indicating the historical probabilities of success or failure (e.g., pharmaceutical R&D’s probability of technical success based on various drug indications can be obtained from external research consulting groups). Finally, Monte Carlo simulation can be run on a model with multiple interacting input assumptions and the output of interest (e.g., net present value, gross margin, tolerance ratios, and development success rates) can be captured as a simulation forecast and the relevant probabilities can be obtained, such as the probability of breaking even, probability of failure, probability of making a profit, and so forth. See Chapter 5 on step-by-step instructions on running and interpreting simulations and probabilities.
Standard Deviation and Variance

Standard deviation is a measure of the average of each data point’s deviation from the mean. A higher standard deviation or variance implies a wider distributional width and, thus, a higher risk.

The standard deviation can be measured in terms of the population or sample, and for illustration purposes, is shown in the following list, where we define $x_i$ as the individual data points, $\mu$ as the population mean, $N$ as the population size, and $n$ as the sample size:

Population standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{N}}$$

and population variance is simply the square of the standard deviation or $\sigma^2$. Alternatively, use Excel’s STDEVP and VARP functions for the population standard deviation and variance respectively.

Sample standard deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

and sample variance is similarly the square of the standard deviation or $s^2$. Alternatively, use Excel’s STDEV and VAR functions for the sample standard deviation and variance respectively. Figure 2.7 shows the step-by-step computations.

The drawbacks of this measure is that both the upside and downside variations are included in the computation of the standard deviation, and its dependence on the units (e.g., values of $x$ in thousands of dollars versus millions of dollars are not comparable). Some analysts define risks as the potential losses or downside; thus, standard deviation and variance penalize upswings as well as downsides. An alternative is the semi-standard deviation.

Semi-Standard Deviation

The semi-standard deviation only measures the standard deviation of the downside risks and ignores the upside fluctuations. Modifications of the semi-standard deviation include calculating only the values below the mean, or values below a threshold (e.g., negative profits or negative cash flows). This
FIGURE 2.7 Standard deviation and variance computation.

<table>
<thead>
<tr>
<th>X</th>
<th>X – Mean</th>
<th>Square of (X – Mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−10.50</td>
<td>−9.07</td>
<td>82.2908</td>
</tr>
<tr>
<td>12.25</td>
<td>13.68</td>
<td>187.1033</td>
</tr>
<tr>
<td>−11.50</td>
<td>−10.07</td>
<td>101.4337</td>
</tr>
<tr>
<td>13.25</td>
<td>14.68</td>
<td>215.4605</td>
</tr>
<tr>
<td>−14.65</td>
<td>−13.22</td>
<td>174.8062</td>
</tr>
<tr>
<td>15.65</td>
<td>17.08</td>
<td>291.6776</td>
</tr>
<tr>
<td>−14.50</td>
<td>−13.07</td>
<td>170.8622</td>
</tr>
</tbody>
</table>

Sum −10.00
Mean −1.43

Population Standard Deviation and Variance

Sum of Square (X – Mean) 1223.6343
Variance = Sum of Square (X – Mean)/N 174.8049
Using Excel’s VARP function: 174.8049
Standard Deviation = Square Root of (Sum of Square (X – Mean)/N) 13.2214
Using Excel’s STDEVP function: 13.2214

Sample Standard Deviation and Variance

Sum of Square (X – Mean) 1223.6343
Variance = Sum of Square (X – Mean)/(N – 1) 203.9390
Using Excel’s VAR function: 203.9390
Standard Deviation = Square Root of (Sum of Square (X – Mean)/(N–1)) 14.2807
Using Excel’s STDEV function: 14.2807

FIGURE 2.7 Standard deviation and variance computation.

approach provides a better picture of downside risk but is more difficult to estimate. Figure 2.8 shows how a sample semi-standard deviation and semi-variance are computed. Note that the computation must be performed manually.

Volatility

The concept of volatility is widely used in the applications of real options and can be defined briefly as a measure of uncertainty and risks. Volatility can be estimated using multiple methods, including simulation of the uncertain variables impacting a particular project and estimating the standard deviation of the resulting asset’s logarithmic returns over time. This concept is more difficult to define and estimate but is more powerful than most other risk measures in that this single value incorporates all sources of uncertainty.
FIGURE 2.8 Semi-standard deviation and semi-variance computation.

rolled into one value. Figure 2.9 illustrates the computation of an annualized volatility. Volatility is typically computed for time-series data only (i.e., data that follows a time series such as stock price, price of oil, interest rates, and so forth). The first step is to determine the relative returns from period to period, take their natural logarithms (ln), and then compute the sample standard deviation of these logged values. The result is the periodic volatility. Then, annualize the volatility by multiplying this periodic volatility by the square root of the number of periods in a year (e.g., 1 if annual data, 4 if quarterly data, and 12 if monthly data are used).

For a more detailed discussion of volatility computation as well as other methods for computing volatility such as using logarithmic present value approach, management assumptions, and GARCH, or generalized autoregressive conditional heteroskedasticity models, and how a discount rate can be determined from volatility, see Real Options Analysis, Second Edition, by Johnathan Mun (Wiley 2005).
Beta is another common measure of risk in the investment finance arena. Beta can be defined simply as the undiversifiable, systematic risk of a financial asset. This concept is made famous through the CAPM, where a higher beta means a higher risk, which in turn requires a higher expected return on the asset. The beta coefficient measures the relative movements of one asset value to a comparable benchmark or market portfolio; that is, we define the beta coefficient as:

\[
\beta = \frac{Cov(x,m)}{Var(m)} = \frac{\rho_{x,m} \sigma_x \sigma_m}{\sigma_m^2}
\]

where \(Cov(x,m)\) is the population covariance between the asset \(x\) and the market or comparable benchmark \(m\), \(Var(m)\) is the population variance of \(m\), where both can be computed in Excel using the \texttt{COVAR} and \texttt{VARP} functions. The computed beta will be for the population. In contrast, the sample beta coefficient is computed using the correlation coefficient between \(x\) and \(m\) or \(\rho_{x,m}\) and the sample standard deviations of \(x\) and \(m\) or using \(s_x\) and \(s_m\) for \(\sigma_x\) and \(\sigma_m\).

A beta of 1.0 implies that the relative movements or risk of \(x\) is identical to the relative movements of the benchmark (see Example 1 in Figure 2.10).
Example 1: Similar fluctuations with the market

<table>
<thead>
<tr>
<th>Months</th>
<th>X</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.50</td>
<td>11.50</td>
</tr>
<tr>
<td>1</td>
<td>12.25</td>
<td>13.25</td>
</tr>
<tr>
<td>2</td>
<td>11.50</td>
<td>12.25</td>
</tr>
<tr>
<td>3</td>
<td>13.25</td>
<td>14.25</td>
</tr>
<tr>
<td>4</td>
<td>14.65</td>
<td>15.65</td>
</tr>
<tr>
<td>5</td>
<td>15.65</td>
<td>16.65</td>
</tr>
<tr>
<td>6</td>
<td>14.50</td>
<td>15.50</td>
</tr>
</tbody>
</table>

**Sample Beta**

- Correlation between X and M using Excel's CORREL: 1.0000
- Standard deviation of X using Excel's STDEV: 1.8654
- Standard deviation of M using Excel's STDEV: 1.8654
- Beta Coefficient: \( \frac{\text{Correlation X and M} \times \text{Stdev X} \times \text{Stdev M}}{\text{Stdev M} \times \text{Stdev M}} \) = 1.0000

**Population Beta**

- Covariance population using Excel's COVAR: 2.9827
- Variance of M using Excel's VARP: 2.9827
- Population Beta: \( \frac{\text{Covariance population (X, M)} / \text{Variance (M)}}{\text{Variance (M)}} \) = 1.0000

Example 2: Half the fluctuations of the market

<table>
<thead>
<tr>
<th>Months</th>
<th>X</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.50</td>
<td>21.00</td>
</tr>
<tr>
<td>1</td>
<td>12.25</td>
<td>24.50</td>
</tr>
<tr>
<td>2</td>
<td>11.50</td>
<td>23.00</td>
</tr>
<tr>
<td>3</td>
<td>13.25</td>
<td>26.50</td>
</tr>
<tr>
<td>4</td>
<td>14.65</td>
<td>29.30</td>
</tr>
<tr>
<td>5</td>
<td>15.65</td>
<td>31.30</td>
</tr>
<tr>
<td>6</td>
<td>14.50</td>
<td>29.00</td>
</tr>
</tbody>
</table>

**Sample Beta**

- Correlation between X and M using Excel's CORREL: 1.0000
- Standard deviation of X using Excel's STDEV: 1.8654
- Standard deviation of M using Excel's STDEV: 3.7308
- Beta Coefficient: \( \frac{\text{Correlation X and M} \times \text{Stdev X} \times \text{Stdev M}}{\text{Stdev M} \times \text{Stdev M}} \) = 0.5000

**Population Beta**

- Covariance population using Excel's COVAR: 5.9653
- Variance of M using Excel's VARP: 11.9306
- Population Beta: \( \frac{\text{Covariance population (X, M)} / \text{Variance (M)}}{\text{Variance (M)}} \) = 0.5000

**FIGURE 2.10** Beta coefficient computation.
where the asset \( x \) is simply one unit less than the market asset \( m \), but they both fluctuate at the same levels). Similarly, a beta of 0.5 implies that the relative movements or risk of \( x \) is half of the relative movements of the benchmark (see Example 2 in Figure 2.10 where the asset \( x \) is simply half the market’s fluctuations \( m \)). Therefore, beta is a powerful measure but requires a comparable to which to benchmark its fluctuations.

**Coefficient of Variation**

The coefficient of variation (CV) is simply defined as the ratio of standard deviation to the mean, which means that the risks are common sized. For example, a distribution of a group of students’ heights (measured in meters) can be compared to the distribution of the students’ weights (measured in kilograms). This measure of risk or dispersion is applicable when the variables’ estimates, measures, magnitudes, or units differ. For example, in the computations in Figure 2.7, the CV for the population is –9.25 or –9.99 for the sample. The CV is useful as a measure of risk per unit of return, or when inverted, can be used as a measure of bang for the buck or returns per unit of risk. Thus, in portfolio optimization, one would be interested in minimizing the CV or maximizing the inverse of the CV.

**Value at Risk**

Value at Risk (VaR) measures the amount of capital reserves at risk given a particular holding period at a particular probability of loss. This measurement can be modified to risk applications by stating, for example, the amount of potential losses a certain percent of the time during the period of the economic life of the project—clearly, a project with a smaller VaR is better. VaR has a holding time period requirement, typically one year or one month. It also has a percentile requirement, for example, a 99.9 percent one-tail confidence. There are also modifications for daily risk measures such as DEaR or Daily Earnings at Risk. The VaR or DEaR can be determined very easily using Risk Simulator; that is, create your risk model, run a simulation, look at the forecast chart, and enter in 99.9 percent as the right-tail probability of the distribution or 0.01 percent as the left tail probability of the distribution, then read the VaR or DEaR directly off the forecast chart.

**Worst-Case Scenario and Regret**

Another simple measure is the value of the worst-case scenario given catastrophic losses. An additional definition is regret; that is, if a decision is made to pursue a particular project, but if the project becomes unprofitable and suffers a loss, the level of regret is simply the difference between the actual losses compared to doing nothing at all. This analysis is very similar
to the VaR but is not time dependent. For instance, a financial return on investment model can be created and a simulation is run. The 5 percent worst-case scenario can be read directly from the forecast chart in Risk Simulator.

**Risk-Adjusted Return on Capital**

Risk-adjusted return on capital (RAROC) takes the ratio of the difference between the fiftieth percentile $P_{50}$ or its median return and the fifth percentile $P_{5}$ return on a project to its standard deviation $\sigma$, written as:

$$RAROC = \frac{P_{50} - P_{5}}{\sigma}$$

This approach is used mostly by banks to estimate returns subject to their risks by measuring only the potential downside effects and truncating the distribution to the worst-case 5 percent of the time, ignoring the positive upswings, while at the same time common sizing to the risk measure of standard deviation. Thus, RAROC can be seen as a measure that combines standard deviation, CV, semi-standard deviation, and worst-case scenario analysis. This measure is useful when applied with Monte Carlo simulation, where the percentiles and standard deviation measurements required can be obtained through the forecast chart’s statistics view in Risk Simulator.

**QUESTIONS**

1. What is the efficient frontier and when is it used?
2. What are inferential statistics and what steps are required in making inferences?
3. When is using standard deviation less desirable than using semi-standard deviation as a measure of risk?
4. If comparing three projects with similar first, second, and fourth moments, would you prefer a project that has no skew, a positive skew, or a negative skew?
5. If comparing three projects with similar first to third moments, would you prefer a project that is leptokurtic (high kurtosis), mesokurtic (average kurtosis), or platykurtic (low kurtosis)? Explain your reasoning with respect to a distribution’s tail area. Under what conditions would your answer change?
6. What are the differences and similarities between Value at Risk and worst-case scenario as a measure of risk?