

# Optimization: Efficient Frontier

Short Examples Series

using

Risk Simulator



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## ***Optimization – Project Portfolio and Efficient Frontier***

**File Name:** *Optimization – Military Portfolio and Efficient Frontier*

**Location:** *Modeling Toolkit | Optimization | Military Portfolio and Efficient Frontier*

**Brief Description:** *This sample model illustrates how to run an optimization on discrete binary integer decision variables in project selection and mix, viewing and interpreting optimization results, creating additional qualitative constraints to the optimization model, and generating an investment Efficient Frontier by applying optimization on changing constraints*

**Requirements:** *Modeling Toolkit, Risk Simulator*

This model shows 20 different projects with different risk-return characteristics as well as several qualitative measures such as strategic score, military readiness score, tactical score, and comprehensive score, and so forth (see Figure 1). These scores are obtained through subject matter experts, for instance, decision makers, leaders, and managers of organizations, where their expert opinions are gathered through the double-blind Delphi Method. After being scrubbed (e.g., extreme values are eliminated, large data variations are analyzed, and multiple iterations of the Delphi Method is performed, and so forth), their respective scores can be entered into a Distributional Fitting routine to find the best-fitting distribution, or used to develop a Custom Distribution for each project.

The central idea of this model is to find the best portfolio allocation such that the portfolio's total comprehensive strategic score and profits are maximized. That is, it is used to find the best project mix in the portfolio that maximizes the total *Profit\*Score* measure, where profit points to the portfolio level net returns after considering the risks and costs of each project, while the Score measures the total comprehensive score of the portfolio, all the while being subject to the constraints of number of projects, budget constraint, full time equivalence resource (FTE) restrictions, and strategic ranking constraints.

### ***Objective:***

*Maximize Total Portfolio Returns times the Portfolio Comprehensive Score (C28)*

### ***Decision Variables:***

*Allocation or Go/No-Go Decision (J5:J24)*

### ***Restrictions on Decision Variables:***

*Binary decision variables (0 or 1)*

### ***Constraints:***

*Total Cost (E26) is less than \$3800 (in thousands or millions of dollars)*

*Less than or equal to 10 projects selected (J26) in the entire portfolio*

*Full-time Equivalence resources have to be less than 80 (M26)*

*Total Strategic Ranking for the entire portfolio must be less than 100 (F28)*

Military Portfolio Optimization												
Project Name	ENPV	NPV	Cost	Strategy Ranking	Return to Rank Ratio	Profitability Index	Selection	Military Score	Tactical Score	FTE Resources	Comprehensive Score	
Project 1	\$458.00	\$150.76	\$1,732.44	1.20	381.67	1.09	1	8.10	2.31	1.20	1.98	
Project 2	\$1,954.00	\$245.00	\$859.00	9.80	199.39	1.29	1	1.27	4.83	2.50	1.76	
Project 3	\$1,599.00	\$458.00	\$1,845.00	9.70	164.85	1.25	1	9.88	4.75	3.60	2.77	
Project 4	\$2,251.00	\$529.00	\$1,645.00	4.50	500.22	1.32	1	8.83	1.61	4.50	2.07	
Project 5	\$849.00	\$564.00	\$458.00	10.90	77.89	2.23	1	5.02	6.25	5.50	2.94	
Project 6	\$758.00	\$135.00	\$52.00	7.40	102.43	3.60	1	3.64	5.79	9.20	3.26	
Project 7	\$2,845.00	\$311.00	\$758.00	19.80	143.69	1.41	1	5.27	6.47	12.50	4.04	
Project 8	\$1,235.00	\$754.00	\$115.00	7.50	164.67	7.56	1	9.80	7.16	5.30	3.63	
Project 9	\$1,945.00	\$198.00	\$125.00	10.80	180.09	2.58	1	5.68	2.39	6.30	2.16	
Project 10	\$2,250.00	\$785.00	\$458.00	8.50	264.71	2.71	1	8.29	4.41	4.50	2.67	
Project 11	\$549.00	\$35.00	\$45.00	4.80	114.38	1.78	1	7.52	4.65	4.90	2.75	
Project 12	\$525.00	\$75.00	\$105.00	5.90	88.98	1.71	1	5.54	5.09	5.20	2.69	
Project 13	\$516.00	\$451.00	\$48.00	2.80	184.29	10.40	1	2.51	2.17	4.60	1.66	
Project 14	\$499.00	\$458.00	\$351.00	9.40	53.09	2.30	1	9.41	9.49	9.90	4.85	
Project 15	\$859.00	\$125.00	\$421.00	6.50	132.15	1.30	1	6.91	9.62	7.20	4.25	
Project 16	\$884.00	\$458.00	\$124.00	3.90	226.67	4.69	1	7.06	9.98	7.50	4.46	
Project 17	\$956.00	\$124.00	\$521.00	15.40	62.08	1.24	1	1.25	2.50	8.60	2.07	
Project 18	\$854.00	\$164.00	\$512.00	21.00	40.67	1.32	1	3.09	2.90	4.30	1.70	
Project 19	\$195.00	\$45.00	\$5.00	1.20	162.50	10.00	1	5.25	1.22	4.10	1.86	
Project 20	\$210.00	\$85.00	\$21.00	1.00	210.00	5.05	1	2.01	4.06	5.20	2.50	
Total	\$22,191.00		\$10,200.44	162.00			20	116.32	97.65	116.60	56.08	
Profit/Rank	\$136.98											
Profit*Score	\$1,244,365.33	Maximize	<= \$3800	<= 100			x <= 10			<= 80		
		Comprehensive Score	Tactical Score	Military Score	Allowed Projects	ROI-RANK Objective						
Budget	\$3,800.00	33.15	62.64	58.58	10	\$470,235.60						
	\$4,800.00	36.33	68.85	66.86	11	\$521,645.92						
	\$5,800.00	38.40	70.46	75.69	12	\$623,557.79						
	\$6,800.00	39.94	72.14	82.31	13	\$659,947.99						
	\$7,800.00	39.76	70.05	86.54	14	\$676,279.81						

Figure 1: The project selection optimization model

### Running an Optimization

To run this preset model, simply open the profile (**Risk Simulator | Change Profile**) and select *Military Portfolio and Efficient Frontier*. Then, run the optimization (**Risk Simulator | Optimization | Run Optimization**) or for practice, set up the model yourself:

1. Start a new profile (**Risk Simulator | New Profile**) and give it a name.
2. In this example, all the allocations are required to be binary (0 or 1) values, so, first select cell **J5** and make this a decision variable in the Integer Optimization worksheet. Select cell **J5** and define it as a decision variable (**Risk Simulator | Optimization | Decision Variables** or click on the *Define Decision icon*) and make it a **Binary Variable**. This setting automatically sets the minimum to 0 and maximum to 1 and can only take on a value of 0 or 1. Then use the **Risk Simulator Copy** on cell **J5**, select cells **J6 to J24**, and use **Risk Simulator's Paste** (**Risk Simulator | Copy Parameter** and **Risk Simulator | Paste Parameter** or use the *Risk Simulator copy and paste icons*, NOT the Excel copy/paste).
3. Next, set up the optimization's constraints by selecting **Risk Simulator | Optimization | Constraints**, and selecting **ADD**. Then link to cell **E26**, and make it **<= 3800**, select **ADD**

- one more time and click on the link icon and point to cell **J26** and set it to  $\leq 10$ . Continue with adding the other constraints (cell **M26**  $\leq 80$  and **F28**  $\leq 100$ ).
4. Select cell **C28**, the objective to be maximized, and select **Risk Simulator | Optimization | Run Optimization**. Review the different tabs to make sure that all the required inputs in steps 2 and 3 above are correct.
  5. You may now select the optimization method of choice and click OK to run the optimization. The model setup is illustrated in Figure 2 and the various optimization procedures are listed next.

***Discrete Optimization** is an optimization that is run on a discrete or static model, where no simulations are run. This optimization type is applicable when the model is assumed to be known and no uncertainties exist. Also, a discrete optimization can be first run to determine the optimal portfolio and its corresponding optimal allocation of decision variables before more advanced optimization procedures are applied. For instance, before running a stochastic optimization problem, a discrete optimization is first run to determine if there exist solutions to the optimization problem before a more protracted analysis is performed.*

***Dynamic Optimization** is applied when Monte Carlo simulation is used together with optimization. Another name for such a procedure is Simulation-Optimization. In other words, a simulation is run for  $N$  trials, and then an optimization process is run for  $M$  iterations until the optimal results are obtained or an infeasible set is found. That is, using Risk Simulator's optimization module, you can choose which forecast and assumption statistics to use and replace in the model after the simulation is run. Then, these forecast statistics can be applied in the optimization process. This approach is useful when you have a large model with many interacting assumptions and forecasts, and when some of the forecast statistics are required in the optimization.*

***Stochastic Optimization** is similar to the dynamic optimization procedure with the exception that the entire dynamic optimization process is repeated  $T$  times. The results will be a forecast chart of each decision variable with  $T$  values. In other words, a simulation is run and the forecast or assumption statistics are used in the optimization model to find the optimal allocation of decision variables. Then, another simulation is run, generating different forecast statistics, and these new updated values are then optimized, and so forth. Hence, the final decision variables will each have its own forecast chart, indicating the range of the optimal decision variables. For instance, instead of obtaining single-point estimates in the dynamic optimization procedure, you can now obtain a distribution of the decision variables, and, hence, a range of optimal values for each decision variable, also known as a stochastic optimization.*

Note: Remember that if you are to run either a dynamic or stochastic optimization routine, make sure that you have assumptions first defined in the model. That is, make sure that some of the cells in C5:C24 and E5:F24 are assumptions. The suggestion for this model is to run a Discrete Optimization.

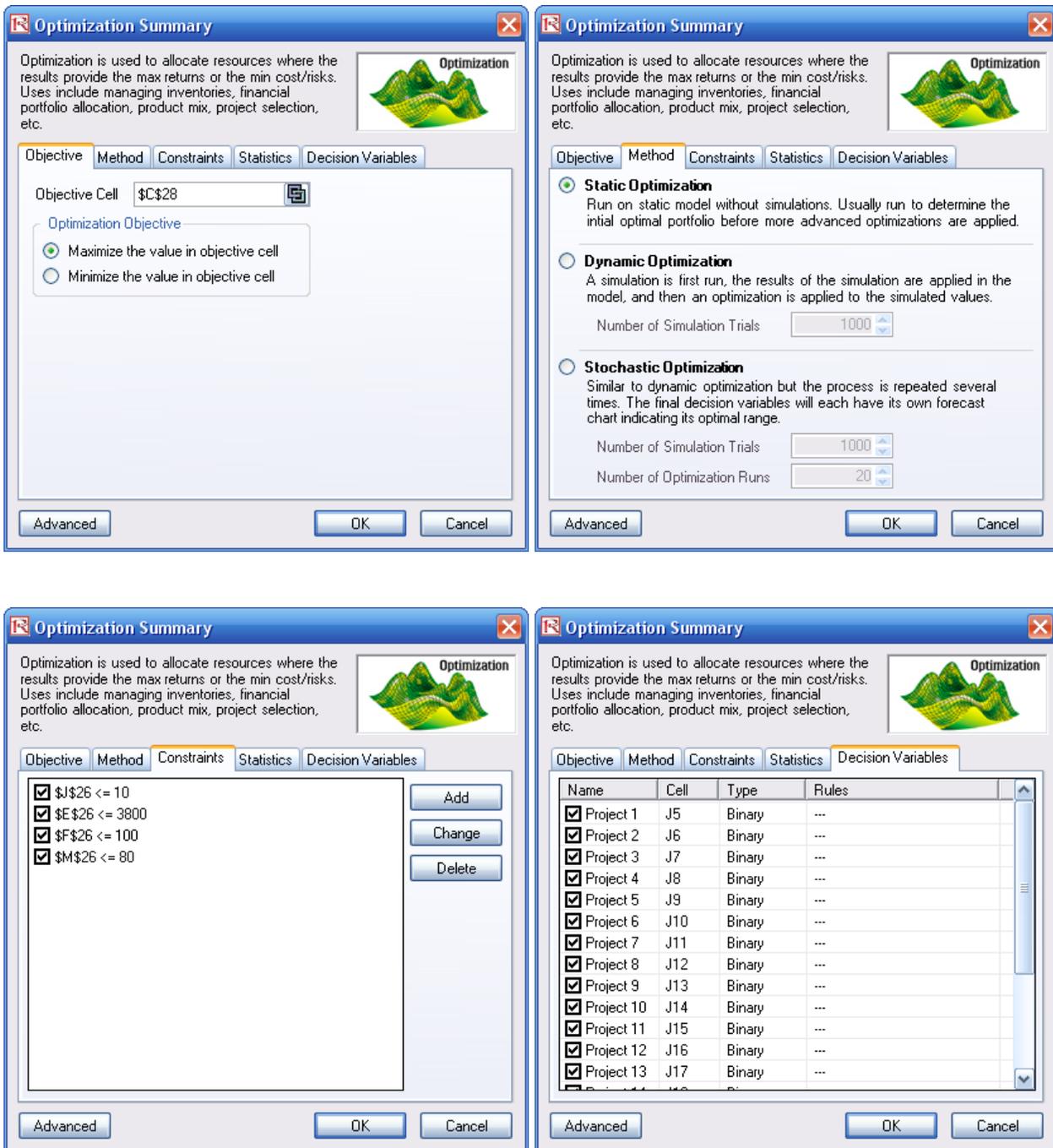


Figure 2: Setting up an optimization model

### Portfolio Efficient Frontier

Clearly, running the optimization procedure will yield an optimal portfolio of projects where the constraints are satisfied. This represents a single optimal portfolio point on the efficient frontier, for example, Portfolio B on the chart in Figure 3. Clearly, by subsequently changing some of the constraints, for instance, by increasing the budget and allowed projects, we can rerun the optimization to produce

another optimal portfolio given these new constraints. Therefore, a series of optimal portfolio allocations can be determined and graphed. This graphical representation of all optimal portfolios is called the Portfolio Efficient Frontier. At this juncture, each point represents a portfolio allocation, for instance, Portfolio B might represent projects: 1, 2, 5, 6, 7, 8, 10, 15, and so forth, while Portfolio C might represent projects 2, 6, 7, 9, 12, 15, and so forth, each resulting in different tactical, military, or comprehensive scores, and portfolio returns. It is up to the decision maker to decide which portfolio represents the best decision and if sufficient resources exist to execute these projects.

Typically, in an Efficient Frontier analysis, you would select projects where the marginal increase in benefits is positive and the slope is steep. In the next example, you would rather select Portfolio D rather than Portfolio E as the marginal increase is negative on the y-axis (e.g., Tactical Score). That is, spending too much money may actually reduce the overall tactical score, and hence this portfolio should not be selected. Also, in comparing Portfolios A and B, you would be more inclined to choose B as the slope is steep and the same increase in budget requirements (x-axis) would return a much higher percentage Tactical Score (y-axis). The decision to choose between Portfolios C and D would depend on available resources and the decision maker deciding if the added benefits warrant and justify the added budget and costs.

<i>Budget</i>	<i>Comprehensive Score</i>	<i>Tactical Score</i>	<i>Military Score</i>	<i>Allowed Projects</i>	<i>ROI-RANK Objective</i>
\$3,800	33.15	62.64	58.58	10	\$470,236
\$4,800	36.33	68.85	66.86	11	\$521,646
\$5,800	38.40	70.46	75.69	12	\$623,558
\$6,800	39.94	72.14	82.31	13	\$659,948
\$7,800	39.76	70.05	86.54	14	\$676,280

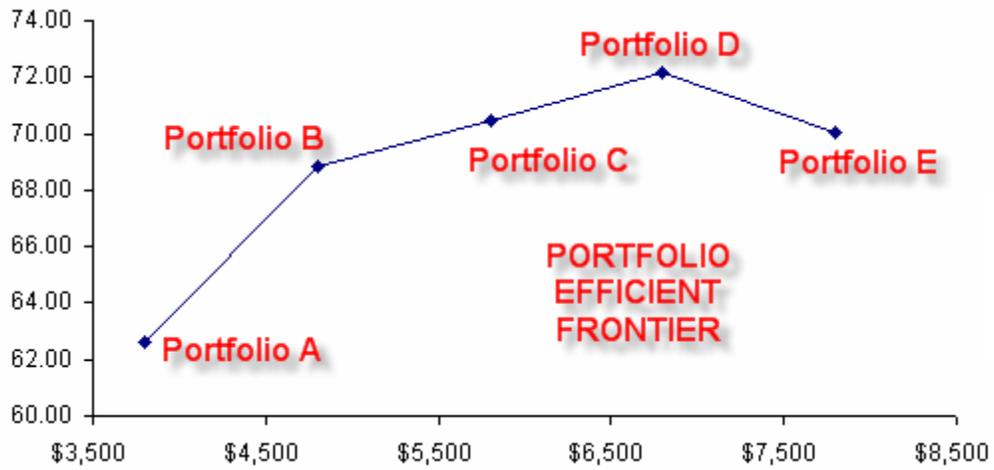


Figure 3: Efficient frontier

To further enhance the analysis, you can obtain the optimal portfolio allocations for C and D and then run a simulation on each optimal portfolio to decide what the probability that D will exceed C in value is, and whether this probability of occurrence justifies the added costs.