Stochastic Forecasting

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In This Issue

1. Become acquanited with the stochastic processes included in Risk Simulator's *Forecasting* tool
2. Learn how to run and interpret a sample stochastic process using Risk Simulator

"How is stochastic process simulation in forecasting?"

Theory

A stochastic process is nothing but a mathematically defined equation that can create a series of outcomes over time, outcomes that are not deterministic in nature; that is, an equation or process that does not follow any simple discernible rule such as price will increase *X*% every year or revenues will increase by this factor of *X* plus *Y*%. A stochastic process is by definition nondeterministic, and one can plug numbers into a stochastic process equation and obtain different results every time. For instance, the path of a stock price is stochastic in nature, and one cannot reliably predict the exact stock price path with any certainty. However, the price evolution over time is enveloped in a process that generates these prices. *The process is fixed and predetermined, but the outcomes are not.* Hence, by stochastic simulation, we create multiple pathways of prices, obtain a statistical sampling of these simulations, and make inferences on the potential pathways that the actual price may undertake given the nature and parameters of the stochastic process used to generate the time series. Four stochastic processes are included in ***Risk Simulator's*** *Forecasting* tool, including Geometric Brownian motion or random walk, which is the most common and prevalently used process due to its simplicity and wide-ranging applications. The other three stochastic processes are the mean-reversion process, jump-diffusion process, and a mixed process.

The interesting thing about stochastic process simulation is that historical data is not necessarily required; that is, the model does not have to fit any sets of historical data. Simply compute the expected returns and the volatility of the historical data or estimate them using comparable external data, or make assumptions about these values.

Procedure

- Start the module by selecting *Risk Simulator* | *Forecasting* | *Stochastic Processes*.
- Select the desired process, enter the required inputs, click on Update Chart a few times to make sure the process is behaving the way you expect it to, and click OK (Figure 1).

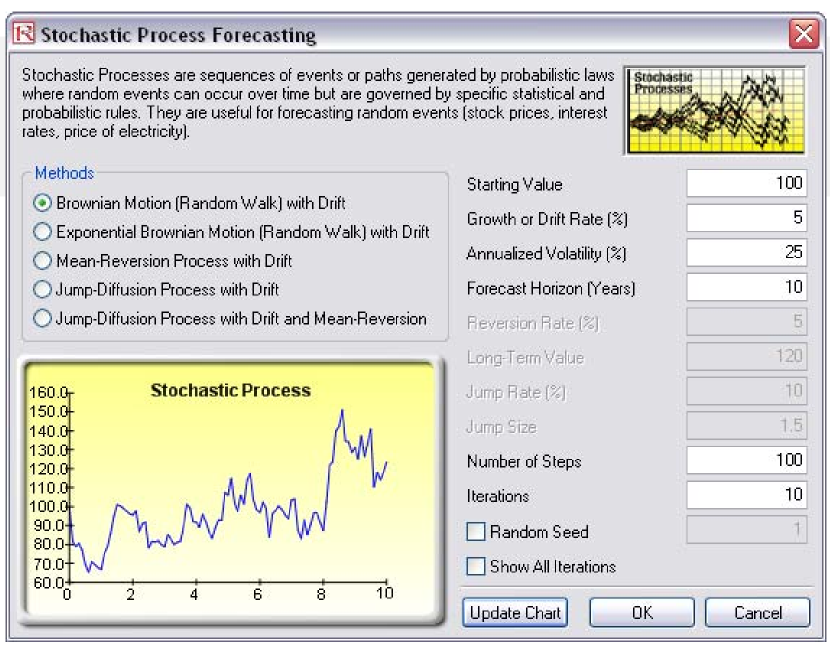

Figure 1. Stochastic Process Forecasting

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Results Interpretation

Figure 2 shows the results of a sample stochastic process. The chart shows a sample set of the iterations while the report explains the basics of stochastic processes. In addition, the forecast values (mean and standard deviation) for each time period is provided. Using these values, you can decide which time period is relevant to your analysis and set assumptions based on these mean and standard deviation values using the normal distribution. These assumptions can then be simulated in your own custom model.

Stochastic Process Forecasting

Statistical Summary

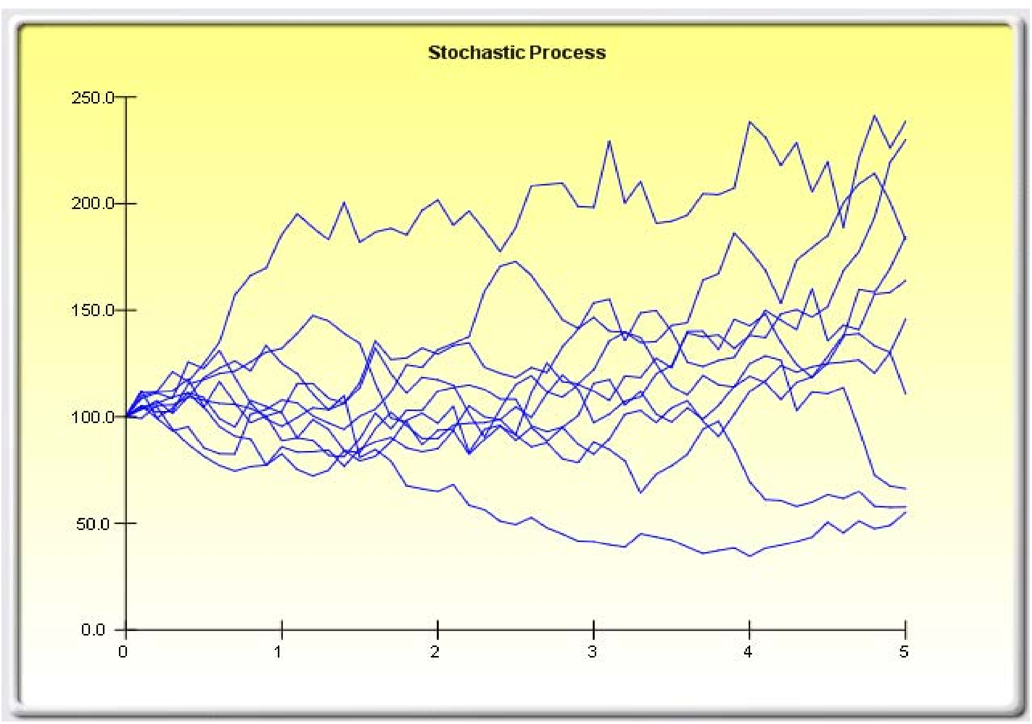
A stochastic process is a sequence of events or paths generated by probabilistic laws. That is, random events can occur over time but are governed by specific statistical and probabilistic rules. The main stochastic processes include Random Walk or Brownian Motion, Mean-Reversion, and Jump-Diffusion. These processes can be used to forecast a multitude of variables that seemingly follow random trends but yet are restricted by probabilistic laws.

The Random Walk Brownian Motion process can be used to forecast stock prices, prices of commodities, and other stochastic time-series data given a drift or growth rate and a volatility around the drift path. The Mean-Reversion process can be used to reduce the fluctuations of the Random Walk process by allowing the path to target a long-term value, making it useful for forecasting time-series variables that have a long-term rate such as interest rates and inflation rates (these are long-term target rates by regulatory authorities or the market). The Jump-Diffusion process is useful for forecasting time-series data when the variable can occasionally exhibit random jumps, such as oil prices or price of electricity (discrete exogenous event shocks can make prices jump up or down). Finally, these three stochastic processes can be mixed and matched as required.

The results on the right indicate the mean and standard deviation of all the iterations generated at each time step. If the Show All Iterations option is selected, each iteration pathway will be shown in a separate worksheet. The graph generated below shows a sample set of the iteration pathways.

Stochastic Process: Brownian Motion (Random Walk) with Drift

Start Value	100	Steps	50.00	Jump Rate	N/A
Drift Rate	5.00%	Iterations	10.00	Jump Size	N/A
Volatility	25.00%	Reversion Rate	N/A	Random Seed	1720050445
Horizon	5	Long-Term Value	N/A		



Time	Mean	Stdev
0.0000	100.00	0.00
0.1000	106.32	4.05
0.2000	105.92	4.70
0.3000	105.23	8.23
0.4000	109.84	11.18
0.5000	107.57	14.67
0.6000	108.63	19.79
0.7000	107.85	24.18
0.8000	109.61	24.46
0.9000	109.57	27.99
1.0000	110.74	30.81
1.1000	111.53	35.05
1.2000	111.07	34.10
1.3000	107.52	32.85
1.4000	108.26	37.38
1.5000	106.36	32.19
1.6000	112.42	32.16
1.7000	110.08	31.24
1.8000	109.64	31.87
1.9000	110.18	36.43
2.0000	112.23	37.63
2.1000	114.32	33.10
2.2000	111.14	38.42
2.3000	111.03	37.69
2.4000	112.04	37.23
2.5000	112.98	40.84
2.6000	115.74	43.69
2.7000	115.11	43.64
2.8000	114.87	43.70
2.9000	113.28	42.25
3.0000	115.72	43.43
3.1000	120.05	50.48
3.2000	116.69	42.61
3.3000	118.31	45.57
3.4000	116.35	40.82
3.5000	115.71	40.33
3.6000	118.69	41.45
3.7000	121.66	45.34
3.8000	121.40	45.03
3.9000	125.19	48.19
4.0000	129.65	55.44
4.1000	129.61	53.82
4.2000	125.86	49.68
4.3000	125.70	53.79
4.4000	126.72	49.70
4.5000	129.52	50.28
4.6000	132.28	49.70
4.7000	138.47	56.77
4.8000	139.69	66.32
4.9000	140.85	65.95
5.0000	143.61	68.65

Figure 2. Stochastic Forecast Result

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Notes

Brownian Motion Random Walk Process

The Brownian motion random walk process takes the form of

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for regular options simulation, or a more generic version takes the form of

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for a geometric process.

For an exponential version, we simply take the exponentials, and as an example, we have

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where we define

S as the variable's previous value

δS as the change in the variable's value from one step to the next

μ as the annualized growth or drift rate

σ as the annualized volatility

To estimate the parameters from a set of time-series data, the drift rate and volatility can be found by setting μ to be the average of the natural logarithm of the relative returns

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while σ is the standard deviation of all

$$ln\frac{S_t}{S_{t-1}}$$

values.

Mean-Reversion Process

The following describes the mathematical structure of a mean-reverting process with drift:

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To obtain the rate of reversion and long-term rate, using the historical data points, run a regression such that

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and we find

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where we define

η as the rate of reversion to the mean

Error! Objects cannot be created from editing field codes. as the long-term value the process reverts to

Y as the historical data series

$\beta_{\square}$ as the intercept coefficient in a regression analysis

$\beta_{\square}$ as the slope coefficient in a regression analysis

Jump Diffusion Process

A jump diffusion process is similar to a random walk process but there is a probability of a jump at any point in time. The occurrences of such jumps are completely random but its probability and magnitude are governed by the process itself as introduced below.

for a jump diffusion process

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where we define

θ as the jump size of S

$F(\lambda)$ as the inverse of the Poisson cumulative probability distribution

λ as the jump rate of S

The jump size can be found by computing the ratio of the postjump to the prejump levels, and the jump rate can be imputed from past historical data. The other parameters are found the same way as above.

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