

QUANTITATIVE RISK ANALYSIS FOR ACTUARIES

**Sample Case Applications of Monte Carlo
Simulation, Real Options Analysis, Stochastic
Forecasting, and Portfolio Optimization**

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Quantitative Risk Analysis for Actuaries¹

Actuaries are typically tasked with analyzing and managing the risks of financial contracts, and an actuary's work is based on the application of mathematical, statistical, economic, and financial analysis applied to a wide range of practical problems in long-term financial planning and management. They act as financial advisors to many commercial organizations such as insurance companies, retirement funds, banks, and stockbrokers. This quick primer is targeted at actuaries to help them better understand the application of various risk-based tools and techniques that will complement their abilities to better serve their clients. The approach is to select a small set of applications from a plethora of possible cases an actuary might come across, and show how the concepts of Monte Carlo simulation, stochastic forecasting, real options analysis, and portfolio optimization can be applied. The methodologies discussed and examples illustrated in this white paper are solved using the *Risk Simulator* software (Monte Carlo simulation, stochastic forecasting, and stochastic optimization) and *Real Options SLS* software (real options analysis and modeling), both by Real Options Valuation, Inc.², and the theoretical discussions are based on the author's several books.

THEORY: Monte Carlo Simulation

Businesses have been dealing with risk since the beginning of the history of commerce. In most cases, managers have looked at the risks of a particular project, acknowledged their existence, and moved on. Little quantification was performed in the past. In fact, most decision makers look only to single-point estimates of a project's profitability. Figure 1 shows an example of a single-point estimate. The estimated net revenue of \$30 is simply that, a single point whose probability of occurrence is close to zero.³ Even in the simple model shown in Figure 1, the effects of interdependencies are ignored, and in traditional modeling jargon, we have the problem of *garbage-in, garbage-out* (GIGO). As an example of interdependencies, the units sold are probably negatively correlated to the price of the product, and positively correlated to the average variable cost, ignoring these effects in a single-point estimate will yield grossly incorrect results. For instance, if the unit sales variable becomes 11 instead of 10, the resulting revenue may not simply be \$35. The net revenue may actually decrease due to an increase in variable cost per unit while the sale price may actually be slightly lower to accommodate this increase in unit sales. Ignoring these interdependencies will reduce the accuracy of the model.

One traditional approach used to deal with risk and uncertainty is the application of scenario analysis. Suppose three scenarios were manufactured: the worst-case, nominal-case, and best-case scenarios, and different values are applied to the unit sales, the resulting three scenarios' net revenues are obtained. As earlier, the problems of interdependencies are not addressed. The net revenues obtained are simply too variable. Not much can be determined from such an analysis.

A related approach is to perform *what-if* or *sensitivity* analysis. Each variable is perturbed a prespecified amount (e.g., unit sales is changed $\pm 10\%$, sales price is changed $\pm 5\%$, and so forth) and the resulting change in net revenues is captured. This approach is great for understanding which variables drive or impact the bottom line the most. Performing such analyses is tedious and provides little benefit at best. A related approach is the use of Monte Carlo simulation and tornado-sensitivity analysis, where all perturbations, scenarios, and sensitivities are run hundreds of thousands of times automatically.

¹ This primer is written by Dr. Johnathan Mun, and is based on his two latest books, "Modeling Risk," (Wiley 2006) and "Real Options Analysis, Second Edition," (Wiley 2005).

² www.realoptionsvaluation.com

³ On a continuous basis, the probability of occurrence is the area under a curve, e.g., there is a 90% probability revenues will be between \$10 and \$11 million. However, the area under a straight line approaches zero. Therefore, the probability of hitting exactly \$10.0000 is close to 0.00000001%.

Therefore, Monte Carlo simulation, one of the advanced concepts introduced in this paper, can be viewed as simply an extension of the traditional approaches of sensitivity and scenario testing. The critical success drivers or the variables that affect the bottom-line variable the most, which at the same time are uncertain, are simulated. In simulation, the interdependencies are accounted for by using correlations. The uncertain variables are then simulated tens of thousands of times automatically to emulate all potential permutations and combinations of outcomes. The resulting net revenues from these simulated potential outcomes are tabulated and analyzed. In essence, in its most basic form, simulation is simply an enhanced version of traditional approaches such as sensitivity and scenario analysis but automatically performed for thousands of times while accounting for all the dynamic interactions between the simulated variables. The resulting net revenues from simulation, as seen in Figure 2, show that there is a 90 percent probability that the net revenues will fall between \$19.44 and \$41.25, with a 5 percent worst-case scenario of net revenues falling below \$19.44. Rather than having only three scenarios, simulation created 5,000 scenarios, or trials, where multiple variables are simulated and changing simultaneously (unit sales, sale price, and variable cost per unit), while their respective relationships or correlations are maintained.

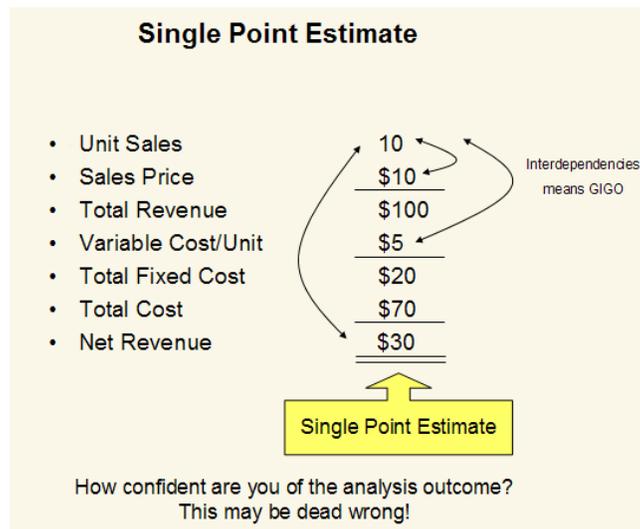


Figure 1: Single-point Estimates

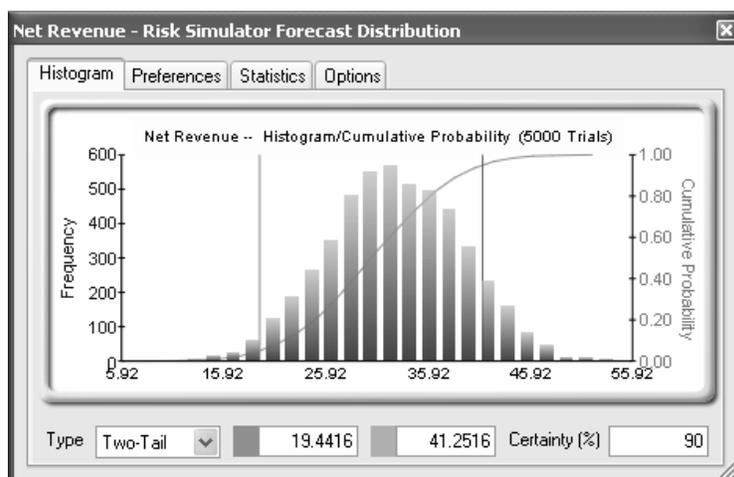


Figure 2: Simulation Results

Monte Carlo simulation, named for the famous gambling capital of Monaco, is a very potent methodology. For the practitioner, simulation opens the door for solving difficult and complex but practical problems with great ease. Perhaps the most famous early use of Monte Carlo simulation was by

the Nobel physicist Enrico Fermi (sometimes referred to as the father of the atomic bomb) in 1930, when he used a random method to calculate the properties of the newly discovered neutron. Monte Carlo methods were central to the simulations required for the Manhattan Project, where in the 1950s Monte Carlo simulation was used at Los Alamos for early work relating to the development of the hydrogen bomb, and became popularized in the fields of physics and operations research. The Rand Corporation and the U.S. Air Force were two of the major organizations responsible for funding and disseminating information on Monte Carlo methods during this time, and today there is a wide application of Monte Carlo simulation in many different fields including engineering, physics, research and development, business, and finance.

Simplistically, Monte Carlo simulation creates artificial futures by generating thousands and even hundreds of thousands of sample paths of outcomes and analyzes their prevalent characteristics. In practice, Monte Carlo simulation methods are used for risk analysis, risk quantification, sensitivity analysis, and prediction. An alternative to simulation is the use of highly complex stochastic closed-form mathematical models. For a high-level decision maker, taking graduate level advanced math and statistics courses is just not logical or practical. A brilliant analyst would use all available tools at his or her disposal to obtain the same answer the easiest and most practical way possible. And in all cases, when modeled correctly, Monte Carlo simulation provides similar answers to the more mathematically elegant methods. In addition, there are many real-life applications where closed-form models do not exist and the only recourse is to apply simulation methods. So, what exactly is Monte Carlo simulation and how does it work?

Today, fast computers and powerful software like *Risk Simulator* have made possible many complex computations that were seemingly intractable in past years. For scientists, engineers, statisticians, managers, business analysts, and others, computers have made it possible to create models that simulate reality and aid in making predictions, one of which is used in simulating real systems by accounting for randomness and future uncertainties through investigating hundreds and even thousands of different scenarios. The results are then compiled and used to make decisions.

Monte Carlo simulation in its simplest form is a random number generator that is useful for forecasting, estimation, and risk analysis. A simulation calculates numerous scenarios of a model by repeatedly picking values from a user-predefined *probability distribution* for the uncertain variables and using those values for the model. As all those scenarios produce associated results in a model, each scenario can have a forecast. Forecasts are events (usually with formulas or functions) that you define as important outputs of the model. Think of the Monte Carlo simulation approach as picking golf balls out of a large basket repeatedly with replacement. The size and shape of the basket depend on the distributional *input assumption* (e.g., a normal distribution with a mean of 100 and a standard deviation of 10, versus a uniform distribution or a triangular distribution) where some baskets are deeper or more symmetrical than others, allowing certain balls to be pulled out more frequently than others. The number of balls pulled repeatedly depends on the number of *trials* simulated. Each ball is indicative of an event, scenario or condition that can occur. For a large model with multiple related assumptions, imagine the large model as a very large basket, where many baby baskets reside. Each baby basket has its own set of colored golf balls that are bouncing around. Sometimes these baby baskets are linked with each other (if there is a *correlation* between the variables), forcing the golf balls to bounce in tandem whereas in other uncorrelated cases, the balls are bouncing independently of one another. The balls that are picked each time from these interactions within the model (the large basket) are tabulated and recorded, providing a *forecast output* result of the simulation.

APPLICATION: Credit Risk Analysis and Private Bond Placements

According to a study conducted by the American Council of Life Insurers and Society of Actuaries, a significant portion of fixed-income securities owned by life insurance companies are private placement bonds. The default-loss experience and economic loss from credit-risk events need to be assessed correctly. The incidence or actual number of occurrence, economic loss or severity of losses (loss actually

sustained given the occurrence of a loss event, as a proportion to the maximum possible losses), economic losses per unit of exposure, and so forth, can be ascertained from historical simulation. For instance, the incidence can be simulated as a Poisson distribution while the losses can be simulated using a variety of distributions, including the lognormal distribution or an exponential distribution. Using historical data, these losses can be statistically and analytically fitted to various distributions to determine the best model, and Monte Carlo simulation can then be applied. Further, some events can be assumed to be independent while others are assumed to be dependent. This dependence can be modeled in Monte Carlo simulation using correlated assumptions in *Risk Simulator*.

In fact, there are a variety of possible applications of simulation in the actuarial world. All example cases going forward in this white paper assumes a Monte Carlo simulation approach run in concert with forecasting, optimization, and real options. In fact, stochastic process forecasting requires Monte Carlo simulation to generate its many random walk and mean-reverting pathways, while real options analysis requires Monte Carlo simulation to obtain an underlying asset's volatility and risk structures, and stochastic optimization uses Monte Carlo simulation to generate different portfolio scenarios and to account for the correlation-diversification effects of multiple projects in a portfolio.

Strategic Real Options Analysis

By applying Monte Carlo simulation to simultaneously change all critical inputs in a correlated manner within a model, you can identify, quantify, and analyze risk.⁴ The question then is what next? Simply quantifying risk is useless unless you can manage it, reduce it, control it, hedge it, or mitigate it. This is where strategic real options analysis comes in. Think of real options as a strategic road map for making decisions. Suppose you are driving from point A to point B, and you only have or know one way to get there, a straight route. Further suppose that there is a lot of *uncertainty* as to what traffic conditions are like further down the road, and you *risk* being stuck in traffic, and there's a 50% chance that will occur. Simulation will provide you the 50% figure. But so what? Knowing that half the time you will get stuck in traffic is valuable information, but the question now is, so what? Especially if you have to get to point B no matter what. However, if you had several alternate routes to get to point B, you can still drive the straight route but if you hit traffic, you can make a left, right, or U-turn, to get around congestion, mitigating the risk, and getting you to point B faster and safer, that is, you have *options*. So, how much is such a strategic road map or global positioning satellite map worth to you? In situations with high risk, real options can help you create strategies to mitigate these risks. In fact, businesses and the military have been doing real options for hundreds of years without really realizing it. For instance, for the military, they call it *courses of action* or *analysis of alternatives*—do we take Hill A so that it provides us the option and ability to take Hill B and Valley C, or do we only take Valley C, and so forth. The piece that is missing is the structure and analytics which real options provides. Using real options analysis, we can quantify and value each strategic pathway, and frame strategies that will hedge or mitigate, and sometimes take advantage of risk.

In the past, corporate investment decisions were cut-and-dried. Buy a new machine that is more efficient, make more products costing a certain amount, and if the benefits outweigh the costs, execute the investment. Hire a larger pool of sales associates, expand the current geographical area, and if the marginal increase in forecast sales revenues exceeds the additional salary and implementation costs, start hiring. Need a new manufacturing plant? Show that the construction costs can be recouped quickly and easily by the increase in revenues it will generate through new and more improved products, and the initiative is approved. However, real-life conditions are a lot more complicated. Your firm decides to go with an e-commerce strategy, but multiple strategic paths exist. Which path do you choose? What are the options that you have? If you choose the wrong path, how do you get back on the right track? How do you value and prioritize the paths that exist? You are a venture capitalist firm with multiple business plans

⁴ The outcomes from a Monte Carlo simulation include probabilities and various risk statistics that can be used to make better decisions.

to consider. How do you value a start-up firm with no proven track record? How do you structure a mutually beneficial investment deal? What is the optimal timing to a second or third round of financing?

Real options are useful not only in valuing a firm through its strategic business options but also as a strategic business tool in capital investment decisions. For instance, should a firm invest millions in a new open architecture initiative, and if so, what are the strategies and how do they proceed? How does the firm choose among several seemingly cashless, costly, and unprofitable information-technology infrastructure projects? Should it indulge its billions in a risky research and development initiative? The consequences of a wrong decision can be disastrous and lives could be at stake. In a traditional analysis, these questions cannot be answered with any certainty. In fact, some of the answers generated through the use of the traditional analysis are flawed because the model assumes a static, one-time decision-making process while the real options approach takes into consideration the strategic options certain projects create under uncertainty and a decision maker's flexibility in exercising or abandoning these options at different points in time, when the level of uncertainty has decreased or has become known over time.

The real options approach incorporates a learning model, such that the decision maker makes better and more informed strategic decisions when some levels of uncertainty are resolved through the passage of time, actions, and events. Traditional analysis assumes a static investment decision, and assumes that strategic decisions are made initially with no recourse to choose other pathways or options in the future. Imagine real options as your guide when navigating through unfamiliar territory, providing road signs at every turn to guide you in making the best and most informed driving decisions. This is the essence of real options. Figure 3 illustrates a very basic real options framing exercise—clearly more complex situations can be set up. From the options that are framed, Monte Carlo simulation, stochastic forecasting, coupled with traditional techniques are applied. Then, real options analytics are applied to solve and value each strategic pathway and an informed decision can then be made.⁵

Real options analysis can be used to frame strategies to mitigate risk, value and find the optimal strategy pathway to pursue, and generate options to enhance the value of the project while managing risks. Sample options include the option to expand, contract, abandon, or sequential compound options (phased stage-gate options, options to wait and defer investments, proof of concept stages, milestone development and research and development initiatives).

APPLICATION: Regime Switching in Long Term Stock Assets

A variable annuity contract with guaranteed living benefits (GLB) is a typical type of long-term liability actuaries deal with. These are typically a modified put option with some money-back guarantee in a mutual fund investment. As these liabilities are linked to equities, historical returns and risks are tabulated and simulation can be used to forecast future scenarios and outcomes. In fact, volatilities can be obtained and simulated to value these options. Rather than relying on a single-point estimate of a Black-Scholes-Merton (BSM) formula, more sophisticated valuation can be performed. The BSM assumes a lognormal equity price distribution with normal returns, executable only at termination. Using real options modeling techniques (closed-form models, simulation based lattices, and so forth), regime changes in volatilities can be modeled with simulation (e.g., jumps between two economic conditions or business structures or market conditions), as well as financial-engineered additions to these GLBs (see the next application example) that might be linked to indexes, profitability of the firm, and so forth. The full distribution function of the option and hence the required asset and capital allocation required to cover these GLB liabilities can be determined.

APPLICATION: Engineering Financial Insurance, Retirement Benefits, Contract Negotiations

More complex pension funds with downside insurance protections (abandonment option) or negotiated settlements and contract payments can be valued using real options approaches. These insurance protect

⁵ The pathways can be valued using partial differential closed-form equations, lattices, and simulation. The Real Options SLS software by Real Options Valuation, Inc. (www.realoptionsvaluation.com) is used to value these options with great ease.

the recipient from market downturns but there is additional cost to the issuer. These instruments' payoffs can be engineered in such a way as to be pegged to some market index, embedded features, or internal firm profitability levels (e.g., if net profits exceed \$100 million, an additional 1% return is provided, or if net profits fall below \$50 million, there will be no payouts, etc.). These obligations can also be used during contract negotiations and payments. For instance, when acquiring a firm, licensing a technology, purchasing a patent, and so forth, instead of up front payments, payments can be made over time, subject to the success of the business, project, or research and development status of the asset. These are sequential compound options, where the acquirer obtains the right to abandon and exit investments, but the ability to continue the next rounds of financing and payments should everything work out, thereby reducing the total risk of failure to the acquirer. The costs of these guarantees can be valued in a real options paradigm.

APPLICATION: Valuing a Firm's Assets as Strategic Options

In trying to value a firm's assets, we can use a real options approach as equity holders essentially have an option to execute the buy out of all bond holders and debt in the company and take over the entire firm. Seeing that equity prices and valuation are easily accessible, as are historical volatilities, we can hence impute, in a real options model, the implied value of the asset.

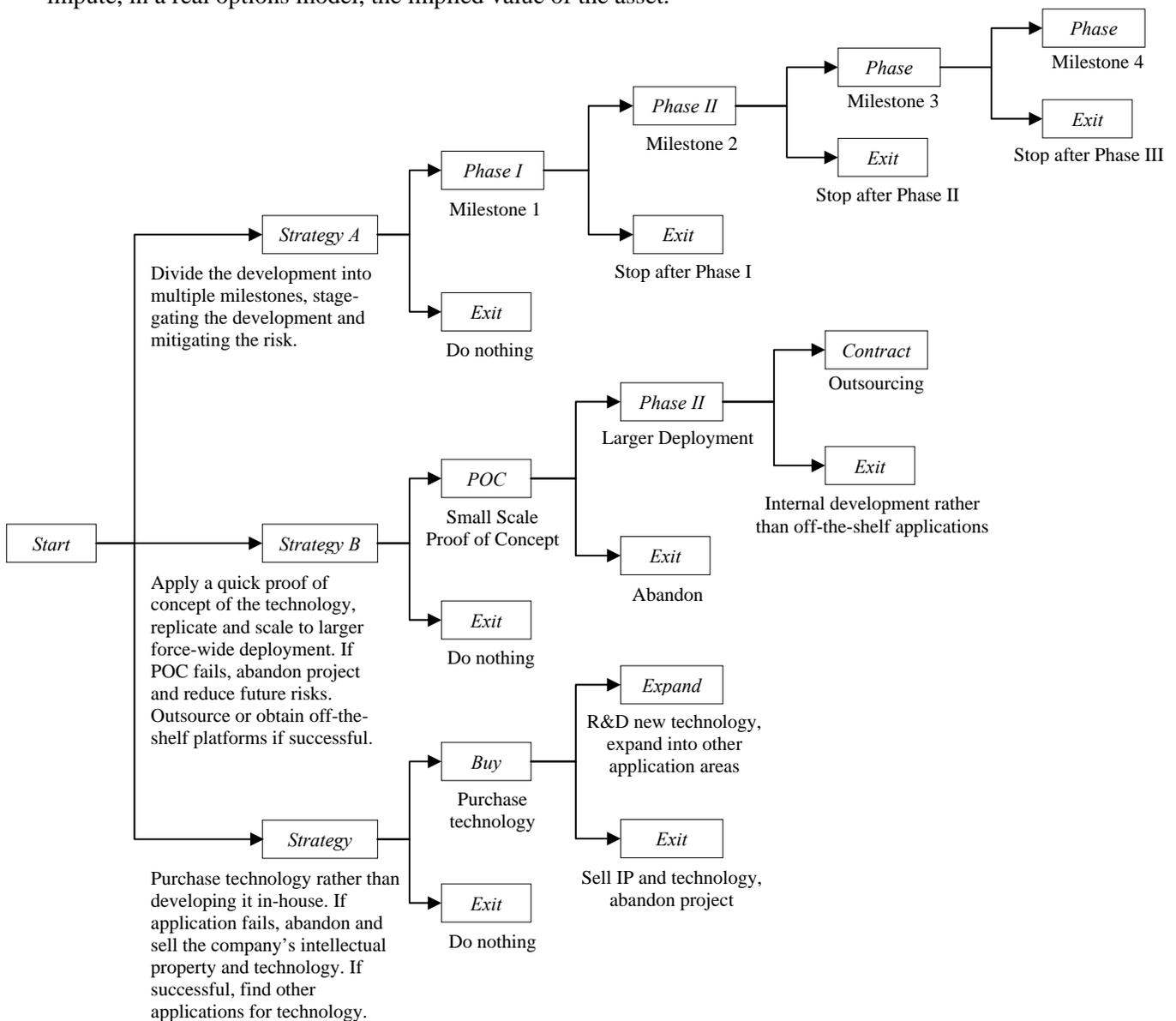


Figure 3: Simple Example of Real Options Framing (Buy vs. Build with Stage-Gate Development)

THEORY: Stochastic Process Forecasting

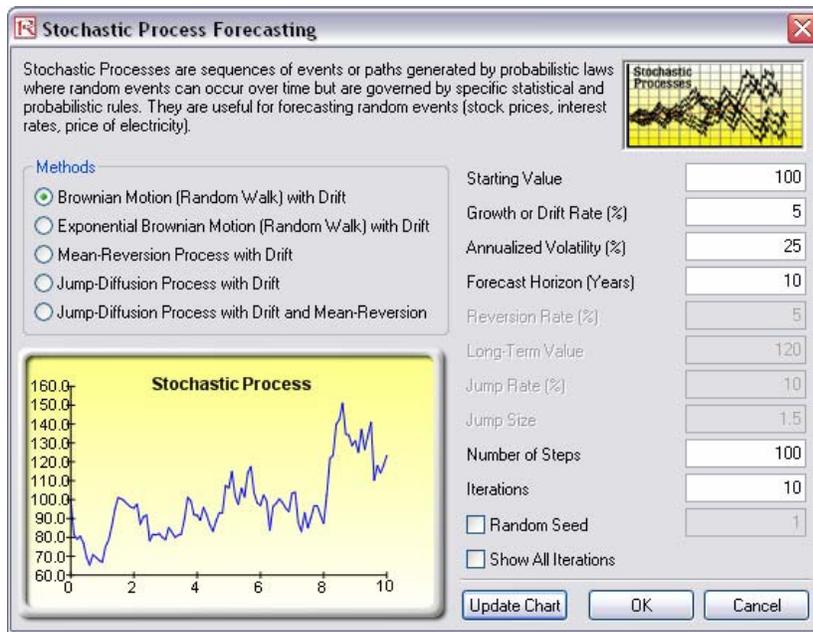
Generally, forecasting can be divided into quantitative and qualitative approaches. Qualitative forecasting is used when little to no reliable historical, contemporaneous, or comparable data exists. Several qualitative methods exist such as the Delphi or expert opinion approach (a consensus-building forecast by field experts, marketing experts, or internal staff members), management assumptions (target growth rates set by senior management), as well as market research or external data or polling and surveys (data obtained through third-party sources, industry and sector indexes, or from active market research). These estimates can be either single-point estimates (an average consensus) or a set of prediction values (a distribution of predictions). The latter can be entered into Risk Simulator as a custom distribution and the resulting predictions can be simulated; that is, running a nonparametric simulation using the prediction data points as the custom distribution.

For quantitative forecasting, the available data or data that needs to be forecasted can be divided into time-series (values that have a time element to them, such as revenues at different years, inflation rates, interest rates, market share, failure rates, and so forth), cross-sectional (values that are time-independent, such as the grade point average of sophomore students across the nation in a particular year, given each student's levels of SAT scores, IQ, and number of alcoholic beverages consumed per week), or mixed panel (mixture between time-series and panel data, e.g., predicting sales over the next 10 years given budgeted marketing expenses and market share projections, which means that the sales data is time-series but exogenous variables such as marketing expenses and market share exist to help to model the forecast predictions).

For actuaries, the most appropriate advanced stochastic forecasting techniques are multivariate regressions and stochastic process forecasting. As regression analysis is a common method, we will not dwell on its technicalities. In contrast, a stochastic process is nothing but a mathematically defined equation that can create a series of outcomes over time, outcomes that are not deterministic in nature; that is, an equation or process that does not follow any simple discernible rule such as price will increase X percent every year or revenues will increase by this factor of X plus Y percent. A stochastic process is by definition nondeterministic, and one can plug numbers into a stochastic process equation and obtain different results every time. For instance, the path of a stock price is stochastic in nature, and one cannot reliably predict the exact stock price path with any certainty. However, the price evolution over time is enveloped in a process that generates these prices. *The process is fixed and predetermined, but the outcomes are not.* Hence, by stochastic simulation, we create multiple pathways of prices, obtain a statistical sampling of these simulations, and make inferences on the potential pathways that the actual price may undertake given the nature and parameters of the stochastic process used to generate the time series. Four stochastic processes are included in Risk Simulator's Forecasting tool, including Geometric Brownian motion or random walk, which is the most common and prevalently used process due to its simplicity and wide-ranging applications. The other three stochastic processes are the mean-reversion process, jump-diffusion process, and a mixed process.

The interesting thing about stochastic process simulation is that historical data is not necessarily required; that is, the model does not have to fit any sets of historical data. Simply compute the expected returns and the volatility of the historical data or estimate them using comparable external data or make assumptions about these values.

Figure 4 shows the results of a sample stochastic process. The chart shows a sample set of the iterations while the report explains the basics of stochastic processes. In addition, the forecast values (mean and standard deviation) for each time period is provided. Using these values, you can decide which time period is relevant to your analysis, and set assumptions based on these mean and standard deviation values using the normal distribution. These assumptions can then be simulated in your own custom model.



Stochastic Process Forecasting

Statistical Summary

A stochastic process is a sequence of events or paths generated by probabilistic laws. That is, random events can occur over time but are governed by specific statistical and probabilistic rules. The main stochastic processes include Random Walk or Brownian Motion, Mean-Reversion, and Jump-Diffusion. These processes can be used to forecast a multitude of variables that seemingly follow random trends but yet are restricted by probabilistic laws.

The Random Walk Brownian Motion process can be used to forecast stock prices, prices of commodities, and other stochastic time-series data given a drift or growth rate and a volatility around the drift path. The Mean-Reversion process can be used to reduce the fluctuations of the Random Walk process by allowing the path to target a long-term value, making it useful for forecasting time-series variables that have a long-term rate such as interest rates and inflation rates (these are long-term target rates by regulatory authorities or the market). The Jump-Diffusion process is useful for forecasting time-series data when the variable can occasionally exhibit random jumps, such as oil prices or price of electricity (discrete exogenous event shocks can make prices jump up or down). Finally, these three stochastic processes can be mixed and matched as required.

The results on the right indicate the mean and standard deviation of all the iterations generated at each time step. If the Show All Iterations option is selected, each iteration pathway will be shown in a separate worksheet. The graph generated below shows a sample set of the iteration pathways.

Stochastic Process: Brownian Motion (Random Walk) with Drift

Start Value	100	Steps	50.00	Jump Rate	N/A
Drift Rate	5.00%	Iterations	10.00	Jump Size	N/A
Volatility	25.00%	Reversion Rate	N/A	Random Seed	1720050445
Horizon	5	Long-Term Value	N/A		

Time	Mean	Stdev
0.0000	100.00	0.00
0.1000	106.32	4.05
0.2000	105.92	4.70
0.3000	105.23	8.23
0.4000	109.84	11.18
0.5000	107.57	14.67
0.6000	108.63	19.79
0.7000	107.85	24.18
0.8000	109.61	24.46
0.9000	109.57	27.99
1.0000	110.74	30.81
1.1000	111.53	35.05
1.2000	111.07	34.10
1.3000	107.52	32.85
1.4000	108.26	37.38
1.5000	106.36	32.19
1.6000	112.42	32.16
1.7000	110.08	31.24
1.8000	109.64	31.87
1.9000	110.18	36.43
2.0000	112.23	37.63
2.1000	114.32	33.10
2.2000	111.14	38.42
2.3000	111.03	37.69
2.4000	112.04	37.23
2.5000	112.98	40.84
2.6000	115.74	43.69
2.7000	115.11	43.64
2.8000	114.87	43.70
2.9000	113.28	42.25
3.0000	115.72	43.43
3.1000	120.05	50.48
3.2000	116.69	42.61
3.3000	118.31	45.57
3.4000	116.35	40.82
3.5000	115.71	40.33
3.6000	118.69	41.45
3.7000	121.66	45.34
3.8000	121.40	45.03
3.9000	125.19	48.19
4.0000	129.65	55.44
4.1000	129.61	53.82
4.2000	125.86	49.68
4.3000	125.70	53.79
4.4000	126.72	49.70
4.5000	129.52	50.28
4.6000	132.28	49.70
4.7000	138.47	56.77
4.8000	139.69	66.32
4.9000	140.85	65.95
5.0000	143.61	68.65

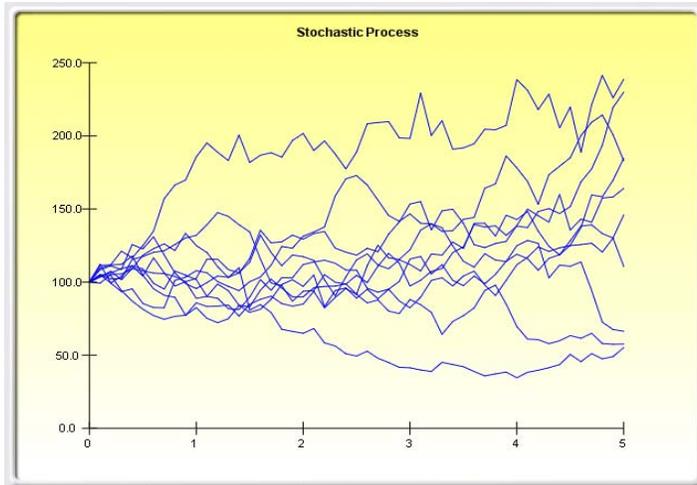


Figure 4: Sample Stochastic Process

APPLICATION: Modeling of Economic Series

In May 2001, the Casualty Actuarial Society and the Society of Actuaries jointly issued requests for research in the areas of modeling and forecasting economic series linked to interest rate scenarios (e.g., interest rates, inflation, unemployment, real estate prices, and so forth). This area of study is very critical and proves to be of significant value and importance to the actuarial profession. These conditions provide the various scenarios under which an insurer's operating decisions rest, as well as assessing their potential impact on corporate value. The stochastic forecasting approaches coupled with dependent and autocorrelated Monte Carlo simulation processes as illustrated in Figure 4 can be applied to test cash flows (for asset-liability matching purposes) across multiple scenarios, assess an insurer's cash position, liquidity levels, solvency testing, capital structures, developing reinsurance structures, and so forth. Fortunately, *Risk Simulator* has a stochastic forecasting module that can generate these forecasts quickly and effortlessly.

Interest rates (nominal and real rates) can be modeled using the Ornstein-Uhlenbeck and Vasicek term-structure model process with a mean-reverting tendency. The following describes the mathematical structure of a mean-reverting process with drift: $\frac{\delta S}{S} = \eta(\bar{S}e^{\mu(\delta t)} - S)\delta t + \mu(\delta t) + \sigma\varepsilon\sqrt{\delta t}$. In order to obtain the rate of reversion and long term rate, using the historical data points, run a regression such that $Y_t - Y_{t-1} = \beta_0 + \beta_1 Y_{t-1} + \varepsilon$ and we find $\eta = -\ln[1 + \beta_1]$ and $\bar{S} = -\beta_0 / \beta_1$.

Where we define

- η as the rate of reversion to the mean
- \bar{S} as the long-term value the process reverts to
- Y as the historical data series
- β as the intercept coefficient in a regression analysis
- β_1 as the slope coefficient in a regression analysis

Other time-series processes such as jump-diffusions can be used to model other economic time-series and price data (e.g., price of oil or electricity). A jump diffusion process is similar to a random walk process but there is a probability of a jump at any point in time. The occurrences of such jumps are completely random but its probability and magnitude are governed by the process itself.

$\frac{\delta S}{S} = \eta(\bar{S}e^{\mu(\delta t)} - S)\delta t + \mu(\delta t) + \sigma\varepsilon\sqrt{\delta t} + \theta F(\lambda)(\delta t)$ for a jump diffusion process.

Where we define

- θ as the jump size of S
- $F(\lambda)$ as the inverse of the Poisson cumulative probability distribution
- λ as the jump rate of S

THEORY: Portfolio Optimization

In most decisions, there are variables over which you have control, such as how much to charge for a product or how much to invest in a project or which projects you should choose in a portfolio when you are constrained by a budget or resources. These controlled variables are called decision variables. Finding the optimal values for decision variables can make the difference between reaching an important goal and missing that goal. These decisions could also include allocating financial resources, building or expanding facilities, managing inventories, and determining product-mix strategies. Such decisions might involve thousands or millions of potential alternatives. Considering and evaluating each of them would be

impractical or even impossible. An optimization model can provide valuable assistance in incorporating relevant variables when analyzing decisions, and finding the best solutions for making decisions. Optimization models often provide insights that intuition alone cannot. An optimization model has three major elements: decision variables, constraints, and an objective. In short, the optimization methodology finds the best combination or permutation of decision variables (e.g., which projects to execute) in every conceivable way such that the objective is maximized (e.g., strategic value, revenues, and return on investment) or minimized (e.g., risk and costs) while still satisfying the constraints (e.g., time, budget, and resources).

Obtaining optimal values generally requires that you search in an iterative or ad hoc fashion. This search involves running one iteration for an initial set of values, analyzing the results, changing one or more values, rerunning the model, and repeating the process until you find a satisfactory solution. This process can be very tedious and time consuming even for small models, and often it is not clear how to adjust the values from one iteration to the next. A more rigorous method systematically enumerates all possible alternatives. This approach guarantees optimal solutions if the model is correctly specified. Suppose that an optimization model depends on only two decision variables. If each variable has 10 possible values, trying each combination requires 100 iterations (10^2 alternatives). If each iteration is very short (e.g., 2 seconds), then the entire process could be done in approximately three minutes of computer time. However, instead of two decision variables, consider six, then consider that trying all combinations requires 1,000,000 iterations (10^6 alternatives). It is easily possible for complete enumeration to take many years to carry out. Therefore, optimization has always been a fantasy until now, with sophisticated software and computing power, coupled with smart heuristics and algorithms, such analyses can be done within minutes.

Figures 5 to 7 illustrate a sample portfolio analysis where in the first case, there are 20 total projects to choose from (if all projects were executed, it would cost \$10.2B), where each project has its own returns on investment or benefits measure, cost, strategic ranking, comprehensive, tactical and total strategic scores (these were obtained from managers and executives through the Delphi method to elicit their thoughts about how strategic a particular project or initiative will be, and so forth). The constraints are full-time equivalence resources, budget, and strategic score. In other words, there are 20 projects or initiatives to choose from, where we want to select the top 10, subject to having enough money to pay for them, the people to do the work, and yet be the most strategic portfolio possible.⁶ All the while, Monte Carlo simulation, real options, and forecasting methodologies are applied in the optimization model (e.g., each project's values shown in Figure 5 are linked from its own large model with simulation and forecasting methodologies applied, and the best strategy for each project is chosen using real options analysis, or perhaps the projects shown are nested within one another, for instance, you cannot exercise Project 2 unless you execute Project 1, but you can only exercise Project 1 without having to do Project 2, and so forth). The results are shown in Figure 5. Figure 6 shows the optimization process done in series, while relaxing some of the constraints. For instance, what would be the best portfolio and the strategic outcome if a budget of \$3.8B was imposed? What if it was increased to \$4.8B, \$5.8B, and so forth? The efficient frontiers depicted in Figure 6 illustrate the best combination and permutation of projects in the optimal portfolio. Each point on the frontier is a portfolio of various combinations of projects that provides the best allocation possible given the requirements and constraints. Finally, Figure 7 shows the top 10 projects that were chosen and how the total budget is best and most optimally allocated to provide the best and most well-balanced portfolio.

⁶ There are 2×10^{18} possible permutations for this problem, and if tested by hand, would take years to complete. Using Risk Simulator, the problem is solved in about 5 seconds, or several minutes if Monte Carlo simulation and real options are incorporated in the analysis.

Project Name	ENPV	NPV	Cost	Strategy Ranking	Return to Rank Ratio	Profitability Index	Selection	Comprehensive Score	Tactical Score	FTE Resources	Strategic Score
Project 1	\$458.00	\$150.76	\$1,732.44	1.20	381.67	1.09	1	8.10	2.31	1.20	1.98
Project 2	\$1,954.00	\$245.00	\$859.00	9.80	199.39	1.29	1	1.27	4.83	2.50	1.76
Project 3	\$1,599.00	\$458.00	\$1,845.00	9.70	164.85	1.25	1	9.88	4.75	3.60	2.77
Project 4	\$2,251.00	\$529.00	\$1,645.00	4.50	500.22	1.32	1	8.83	1.61	4.50	2.07
Project 5	\$849.00	\$564.00	\$458.00	10.90	77.89	2.23	1	5.02	6.25	5.50	2.94
Project 6	\$758.00	\$135.00	\$52.00	7.40	102.43	3.60	1	3.64	5.79	9.20	3.26
Project 7	\$2,845.00	\$311.00	\$758.00	19.80	143.69	1.41	1	5.27	6.47	12.50	4.04
Project 8	\$1,235.00	\$754.00	\$115.00	7.50	164.67	7.56	1	9.80	7.16	5.30	3.63
Project 9	\$1,945.00	\$198.00	\$125.00	10.80	180.09	2.58	1	5.68	2.39	6.30	2.16
Project 10	\$2,250.00	\$785.00	\$458.00	8.50	264.71	2.71	1	8.29	4.41	4.50	2.67
Project 11	\$549.00	\$35.00	\$45.00	4.80	114.38	1.78	1	7.52	4.65	4.90	2.75
Project 12	\$525.00	\$75.00	\$105.00	5.90	88.98	1.71	1	5.54	5.09	5.20	2.69
Project 13	\$516.00	\$451.00	\$48.00	2.80	184.29	10.40	1	2.51	2.17	4.60	1.66
Project 14	\$499.00	\$458.00	\$351.00	9.40	53.09	2.30	1	9.41	9.49	9.90	4.85
Project 15	\$859.00	\$125.00	\$421.00	6.50	132.15	1.30	1	6.91	9.62	7.20	4.25
Project 16	\$884.00	\$458.00	\$124.00	3.90	226.67	4.69	1	7.06	9.98	7.50	4.46
Project 17	\$956.00	\$124.00	\$521.00	15.40	62.08	1.24	1	1.25	2.50	8.60	2.07
Project 18	\$854.00	\$164.00	\$512.00	21.00	40.67	1.32	1	3.09	2.90	4.30	1.70
Project 19	\$195.00	\$45.00	\$5.00	1.20	162.50	10.00	1	5.25	1.22	4.10	1.86
Project 20	\$210.00	\$85.00	\$21.00	1.00	210.00	5.05	1	2.01	4.06	5.20	2.50
Total	\$22,191.00		\$10,200.44	162.00			20	116.32	97.65	116.60	56.08
Profit/Rank	\$136.98										
Profit*Score	\$1,244,365.33	Maximize	< = \$3800	< = 100			x <= 10			< = 80	

Figure 5: Portfolio Optimization and Allocation

Budget	Strategic Score	Tactical Score	Comprehensive Score	Allowed Projects	ROI-RANK Objective
\$3,800.00	33.15	62.64	58.58	10	\$470,235.60
\$4,800.00	36.33	68.85	66.86	11	\$521,645.92
\$5,800.00	38.40	70.46	75.69	12	\$623,557.79
\$6,800.00	39.94	72.14	82.31	13	\$659,947.99
\$7,800.00	39.76	70.05	86.54	14	\$676,279.81

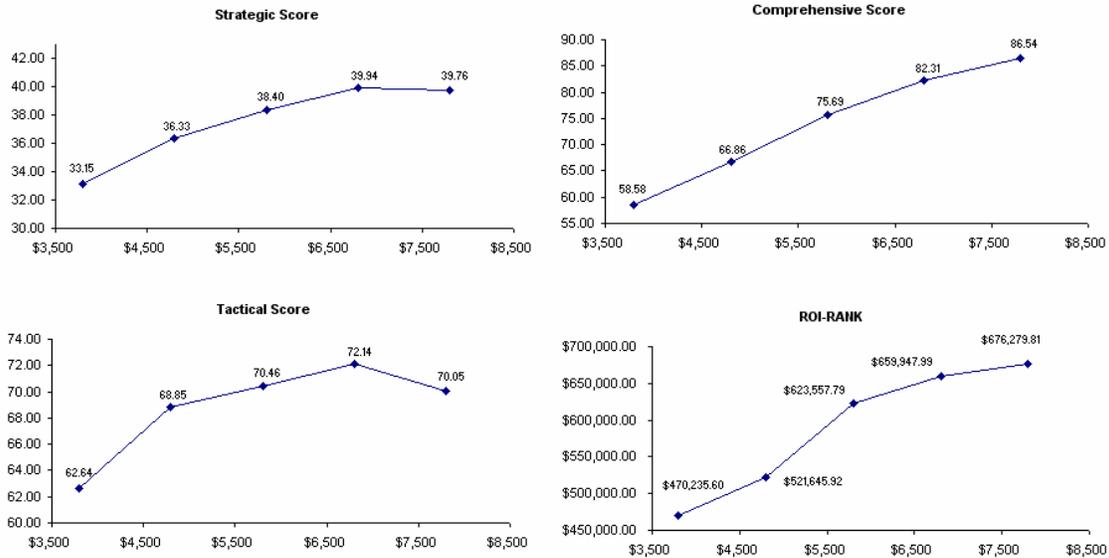


Figure 6: Efficient Frontiers of Portfolios

ASSET ALLOCATION OPTIMIZATION MODEL

Asset Class Description	Annualized Returns	Volatility Risk	Allocation Weights	Required Minimum Allocation	Required Maximum Allocation	Return to Risk Ratio	Returns Ranking (Hi-Lo)	Risk Ranking (L-Hi)	Return to Risk Ranking (Hi-Lo)	Allocation Ranking (Hi-Lo)
Selected Project 1	10.50%	12.38%	11.10%	5.00%	35.00%	0.8483	9	2	7	4
Selected Project 2	11.12%	16.36%	6.74%	5.00%	35.00%	0.6799	7	8	10	10
Selected Project 3	11.77%	15.81%	7.63%	5.00%	35.00%	0.7445	6	7	9	9
Selected Project 4	10.77%	12.33%	11.49%	5.00%	35.00%	0.8738	8	1	5	3
Selected Project 5	13.49%	13.35%	12.26%	5.00%	35.00%	1.0102	5	4	2	2
Selected Project 6	14.24%	14.53%	10.94%	5.00%	35.00%	0.9800	3	6	3	5
Selected Project 7	15.60%	14.30%	12.36%	5.00%	35.00%	1.0908	1	5	1	1
Selected Project 8	14.95%	16.64%	8.75%	5.00%	35.00%	0.8983	2	10	4	7
Selected Project 9	14.15%	16.56%	8.36%	5.00%	35.00%	0.8545	4	9	6	8
Selected Project 10	10.08%	12.55%	10.37%	5.00%	35.00%	0.8027	10	3	8	6
Portfolio Total	12.7270%	4.54%	100.00%							
Return to Risk Ratio	2.8021									

Figure 7: Portfolio Optimization (Continuous Allocation of Funds)

APPLICATION: Allocation of Required Capital Across Multiple Business Lines

Insurance companies in particular, have various methods to determine the required capital mix and allocation across different businesses and risks. For instance, the NAIC RBC (National Association of Insurance Commissioners' Risk-Based Capital) approach recommends that risks from various business lines be separated into four broad categories (asset risk, insurance risk, interest rate risk, and general business risk, with the potential for even greater sub-corporate segregation). Their model is a static distribution of weights to obtain the efficient allocation of capital mix in an insurance company. Specifically, we have the allocation to weights equity-like assets (γ), insurance risk (ε), health insurance and credit risk (δ), and interest rate risk (β) where each weight (w) is defined as $w_i = \frac{\text{risk type}}{\Omega}$, and where

$\Omega = \sqrt{(\alpha + \beta)^2 + \gamma^2 + \varepsilon^2 + \delta^2 + \phi^2}$. Here, α is the non-equity asset risk, and ϕ is the health insurance administrative expense risk. The cross terms between α and β accounts for some relationship in the model but in general, is a *deterministic* model with predefined outcomes regardless of the risk or uncertainty of the inputs, and is solved using a mean-variance partial differential approach. Ω is simply the sums of squares of the risk elements in the organization, and the allocation weights are the risk levels of each component as a percent of the total sums of squares. In contrast, Figure 7 shows a more dynamic model where the capital allocation can be determined analytically. In addition, each of the inputs in Figure 7, namely, the returns and risk levels, are all stochastic and simulated. Therefore, Monte Carlo simulation based on fitting the distributions to historical data can be run to model the risks and returns of each business line, and optimization can then be applied to determine the efficient capital allocation across these multiple business lines. This way, cross-correlations, nonlinear relationships, different statistical distributions of risks and returns, cannibalization, and any other analytical factors can be considered. The result is the best and most efficient allocation of capital.

APPLICATION: Solvency Requirements and Tail VaR

Another application is the allocation of solvency capital across multiple lines of business, using the Risk-Tail VaR (Value at Risk) extension. Globally, the International Accounting Standards Board, International Association of Insurance Supervisors, the International Actuarial Association, the Basel Committee, and Bank of International Settlements have been developing standards of capital requirements. They typically employ the Tail VaR concept for say, 99.95% confidence that the enterprise does not become technically insolvent. The probability distributions of potential losses are dependent on one another, and cross-correlations may create diversification effects. The use of simple normal distributions or lognormal assumptions to simplify the math involved is ill advised here, as simplifications in such a manner can only be used as a benchmark and is not robust. Rather, historical simulation with curve fitting and a correlation matrix can be applied in Monte Carlo simulation in concert with portfolio

optimization. Figure 8 shows the results from a correlated simulation within an equally distributed portfolio of 10 business lines. The naïve Tail VaR excess capital required is \$37.01 million. However, after considering the relative uncertainties of the mean required capital and each business line’s cross-correlation hedging effects, the required capital is now only \$13.68 million at the VaR 99.95 percentile, indicating significant benefits gained by pooling risks. Instead of relying on a static mean-variance optimization procedure for computing portfolio volatility where we have $\sigma_p = \sqrt{w_i^2 \sigma_i^2 + 2\rho_{ij} \sigma_i \sigma_j}$, and projecting the theoretical distribution’s Tail VaR using some arbitrary distribution (e.g., normal or lognormal), we can now analytically and empirically using historical loss data, run correlated Monte Carlo simulations and obtain the empirically-based Tail VaR which provides a much more sophisticated, robust, and analytically more accurate, as well as lower capital requirement.

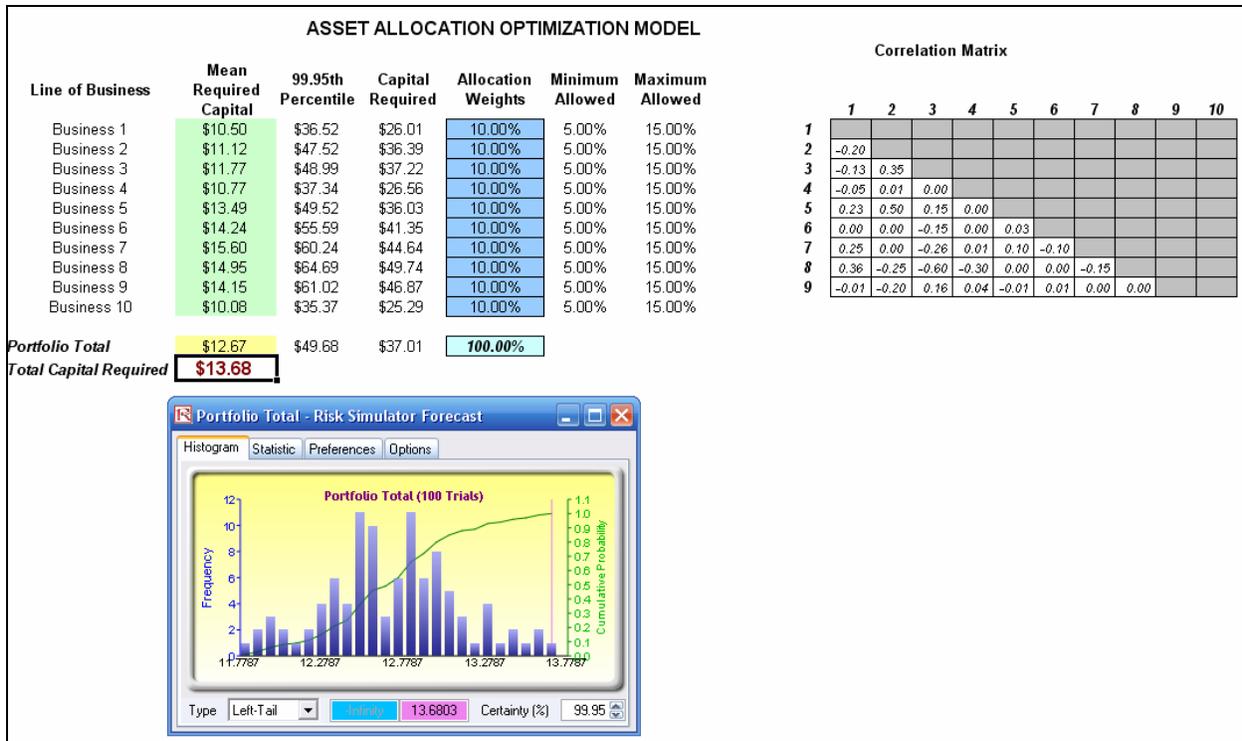


Figure 8: Correlated Portfolio’s Tail VaR Capital Requirements

Integrated Risk Analysis Framework

We are now able to put all the pieces together into an *integrated risk analysis framework* and see how these different techniques are related in a risk analysis and risk management context. This framework comprises eight distinct phases of a successful and comprehensive risk analysis implementation, going from a qualitative management screening process to creating clear and concise reports for management. The process was developed by the author based on previous successful implementations of risk analysis, forecasting, real options, valuation, and optimization projects both in the consulting arena and in industry-specific problems. These phases can be performed either in isolation or together in sequence for a more robust integrated analysis.

Figure 9 shows the integrated risk analysis process up close. We can segregate the process into the following eight simple steps:

1. Qualitative management screening.
2. Time-series and regression forecasting.
3. Base case KVA and net present value analysis.
4. Monte Carlo simulation.
5. Real options problem framing.
6. Real options modeling and analysis.
7. Portfolio and resource optimization.
8. Reporting and update analysis.

1. Qualitative Management Screening

Qualitative management screening is the first step in any integrated risk analysis process. Decision makers have to decide which projects, assets, initiatives, or strategies are viable for further analysis, in accordance with the organization's mission, vision, goal, or overall business strategy. The organization's mission, vision, goal, or overall business strategy may include strategies and tactics, competitive advantage, technical, acquisition, growth, synergistic, or globalization issues. That is, the initial list of projects should be qualified in terms of meeting the decision maker's agenda. Often the most valuable insight is created as decision makers frame the complete problem to be resolved. This is where the various risks to the organization are identified and flushed out.

2. Time-Series and Regression Forecasting

The future is then forecasted using time-series analysis, stochastic forecasting, or multivariate regression analysis if historical or comparable data exist. Otherwise, other qualitative forecasting methods may be used (subjective guesses, growth rate assumptions, expert opinions, Delphi method, and so forth).⁷ *Risk Simulator* can also be used to run more advanced forecasting techniques such as nonlinear extrapolation, stochastic processes (mean-reversion, random walk, jump-diffusion, and mixed processes) as well as Box-Jenkins ARIMA econometric models.

3. Base Case Net Present Value Analysis

For each project that passes the initial qualitative screens a discounted cash flow model is created. This model serves as the base case analysis where a net present value or ROI is calculated for each project, using the forecasted values in the previous step. This step also applies if only a single project is under evaluation. This net present value is calculated using the traditional approach of using the forecast revenues and costs, and discounting the net of these revenues and costs at an appropriate risk-adjusted rate. The return on investment and other metrics are generated here. For nonprofit organizations, governmental, or military organizations, we can also apply Knowledge Value-Added (KVA) analysis in this phase. KVA provides the required *benefits* or revenue proxy to run ROI analysis. KVA measures the value provided by human capital assets and IT assets by analyzing an organization, process or function at the process-level. It provides insights into each dollar of IT investment by monetizing the outputs of all assets, including intangible assets (e.g., such as that produced by IT and humans). By capturing the value of knowledge embedded in an organization's core processes (i.e., employees and IT), KVA identifies the actual cost and revenue of a process, product, or service. Because KVA identifies every process required to produce an aggregated output in terms of the historical prices and costs per common unit of output of those processes, unit costs and unit prices of can be calculated. The methodology has been applied in 45 areas within the Department of Defense, from flight scheduling applications to ship maintenance and modernization processes.

⁷ See Chapters 8 and 9 of "Modeling Risk," (Wiley 2006) by Dr. Johnathan Mun for details on forecasting and using the author's *Risk Simulator* software to run time-series, extrapolation, stochastic process, ARIMA, and regression forecasts.

4. Monte Carlo Simulation⁸

Because the static discounted cash flow produces only a single-point estimate result, there is oftentimes little confidence in its accuracy given that future events that affect forecast cash flows are highly uncertain. To better estimate the actual value of a particular project, Monte Carlo simulation should be employed next. Usually, a sensitivity analysis is first performed on the discounted cash flow model; that is, setting the net present value or ROI as the resulting variable, we can change each of its precedent variables and note the change in the resulting variable. Precedent variables include revenues, costs, tax rates, discount rates, capital expenditures, depreciation, and so forth, which ultimately flow through the model to affect the net present value or ROI figure. By tracing back all these precedent variables, we can change each one by a preset amount and see the effect on the resulting net present value. A graphical representation can then be created in Risk Simulator, which is oftentimes called a tornado chart because of its shape, where the most sensitive precedent variables are listed first, in descending order of magnitude. Armed with this information, the analyst can then decide which key variables are highly uncertain in the future and which are deterministic. The uncertain key variables that drive the net present value and hence, the decision are called critical success drivers. These critical success drivers are prime candidates for Monte Carlo simulation. Because some of these critical success drivers may be correlated, a correlated and multidimensional Monte Carlo simulation may be required. Typically, these correlations can be obtained through historical data. Running correlated simulations provides a much closer approximation to the variables' real-life behaviors.

5. Real Options Problem Framing⁹

The question now is that after quantifying risks in the previous step, what next? The risk information obtained somehow needs to be converted into *actionable intelligence*. Just because risk has been quantified to be such and such using Monte Carlo simulation, so what and what do we do about it? The answer is to use real options analysis to hedge these risks, to value these risks, and to position yourself to take advantage of the risks. The first step in real options is to generate a strategic map through the process of framing the problem. Based on the overall problem identification occurring during the initial qualitative management screening process, certain strategic optionalities would have become apparent for each particular project. The strategic optionalities may include among other things, the option to expand, contract, abandon, switch, choose, and so forth. Based on the identification of strategic optionalities that exist for each project or at each stage of the project, the analyst can then choose from a list of options to analyze in more detail. Real options are added to the projects to hedge downside risks and to take advantage of upside swings.

In this phase, complex systems modeling, system dynamics, and game theory can also be applied. That is, the model can be set up as a system of multiple links or nested options, where one option is linked to another in unison (e.g., complex simultaneous and sequential compound options) and the ramifications of competitors' actions can be included in the model (e.g., strategic competitive games played in an oligopoly situation, where your opponent takes very different actions depending on the actions that you take, generating multiple potential scenarios and payoffs).

6. Real Options Modeling and Analysis

Through the use of Monte Carlo simulation, the resulting stochastic discounted cash flow model will have a distribution of values. Thus, simulation models, analyzes, and quantifies the various risks and uncertainties of each project. The result is a distribution of the NPVs and the project's volatility. In real options, we assume that the underlying variable is the future profitability of the project, which is the future cash flow series. An implied volatility of the future free cash flow or underlying variable can be calculated through the results of a Monte Carlo simulation previously performed. Usually, the volatility is

⁸ See Chapters 4 and 5 of "Modeling Risk," (Wiley 2006) by Dr. Johnathan Mun for details on running Monte Carlo simulation using the author's *Risk Simulator* software.

⁹ See "Real Options Analysis, Second Edition: Tools and Techniques," (Wiley 2005) by Dr. Johnathan Mun for more technical details on framing and solving real options problems.

measured as the standard deviation of the logarithmic returns on the free cash flow stream. In addition, the present value of future cash flows for the base case discounted cash flow model is used as the initial underlying asset value in real options modeling. Using these inputs, real options analysis is performed to obtain the projects' strategic option values.

7. Portfolio and Resource Optimization¹⁰

Portfolio optimization is an optional step in the analysis. If the analysis is done on multiple projects, decision makers should view the results as a portfolio of rolled-up projects because the projects are in most cases correlated with one another, and viewing them individually will not present the true picture. As organizations do not only have single projects, portfolio optimization is crucial. Given that certain projects are related to others, there are opportunities for hedging and diversifying risks through a portfolio. Because firms have limited budgets, have time and resource constraints, while at the same time have requirements for certain overall levels of returns, risk tolerances, and so forth, portfolio optimization takes into account all these to create an optimal portfolio mix. The analysis will provide the optimal allocation of investments across multiple projects.

8. Reporting and Update Analysis

The analysis is not complete until reports can be generated. Not only are results presented, but the process should also be shown. Clear, concise, and precise explanations transform a difficult black-box set of analytics into transparent steps. Decision makers will never accept results coming from black boxes if they do not understand where the assumptions or data originate and what types of mathematical or analytical massaging takes place. Risk analysis assumes that the future is uncertain and that decision makers have the right to make midcourse corrections when these uncertainties become resolved or risks become known; the analysis is usually done ahead of time and thus, ahead of such uncertainty and risks. Therefore, when these risks become known over the passage of time, actions, and events, the analysis should be revisited to incorporate the decisions made or revising any input assumptions. Sometimes, for long-horizon projects, several iterations of the real options analysis should be performed, where future iterations are updated with the latest data and assumptions. Understanding the steps required to undertake an integrated risk analysis is important because it provides insight not only into the methodology itself but also into how it evolves from traditional analyses, showing where the traditional approach ends and where the new analytics start. In this phase, management dashboards, balanced scorecards, and other reporting mechanisms can be created from the outputs of the previous seven phases.

Conclusion

Hopefully it has now become evident that the decisions of the future require the use of more advanced analytical procedures for making strategic investment decisions and when managing portfolios of projects. In the past, due to the lack of technological maturity, this would have been extremely difficult, and hence businesses had to resort to experience and managing by gut feel. Nowadays with the assistance of technology and more mature methodologies, there is simply no excuse not to take the analysis a step further. Corporations like 3M, Airbus, Boeing, BP, Chevron, Johnson & Johnson, Motorola, Pfizer, and many others have already been successfully using these techniques for years. The relevant software applications, books, case studies, and public seminars have all been created and proof of concept case studies have already been developed.¹¹ The only critical barrier to implementation, simply put, is the lack of education or exposure. Many have not seen or even heard of these new concepts and hopefully this primer, if it was successful, serves to open the eyes of the reader to a wealth of analytical techniques and tools out there that can complement what is currently being done.

¹⁰ See Chapters 10 and 11 of "Modeling Risk," (Wiley 2006) by Dr. Johnathan Mun for details on using Risk Simulator to perform portfolio optimization.

¹¹ See www.realoptionsvaluation.com (Download site) for more details on the software applications Risk Simulator and Real Options SLS as well as sample case studies, videos, sample models, and training seminars (e.g., the 4-day Certified Risk Analyst public seminars cover all the methodologies outlined in this primer and more).

Integrated Risk Management Process

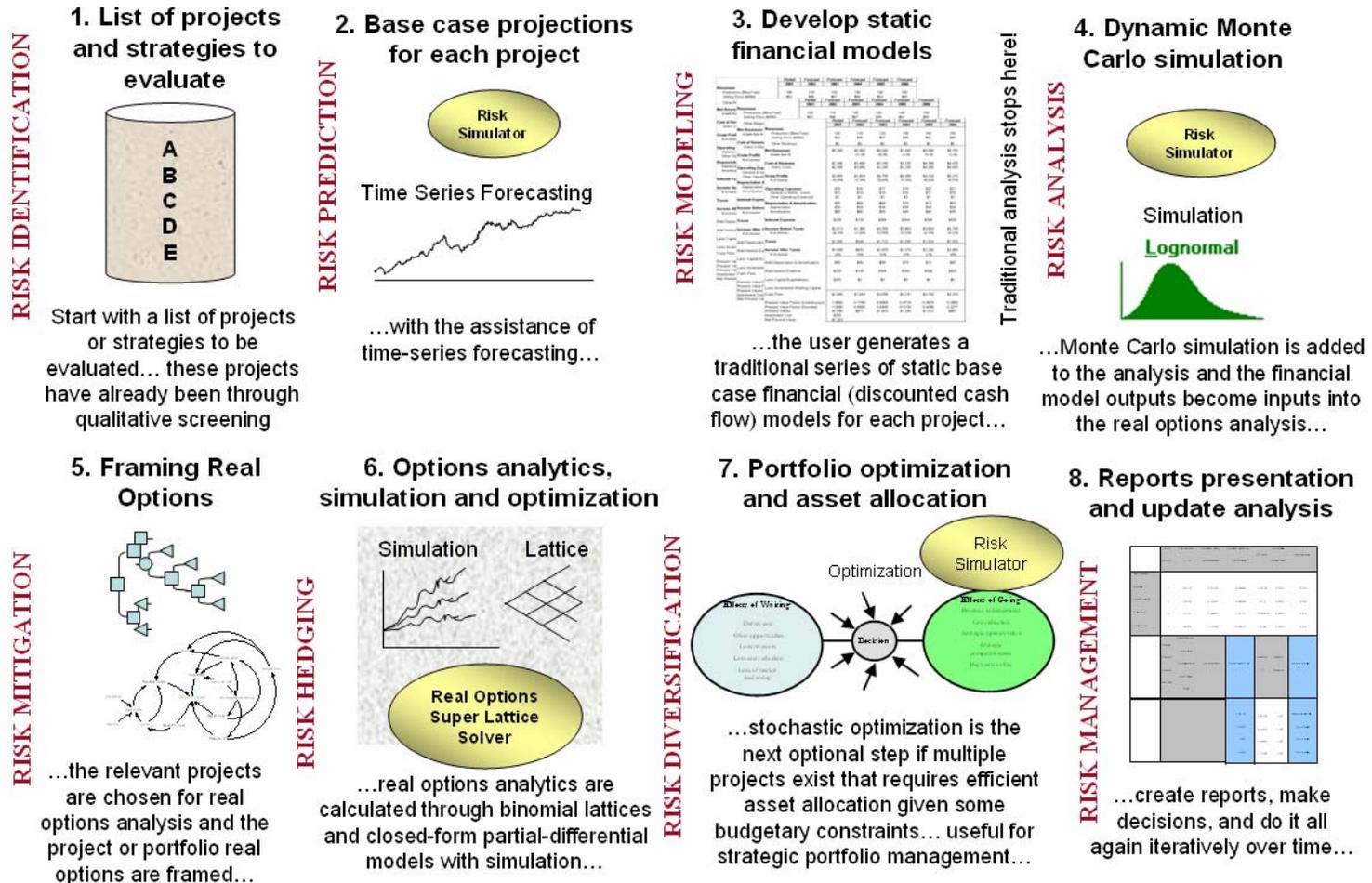


Figure 9 – Integrated Risk Management Process

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